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STRUCTURAL BEHAVIOR OF PRESSURE-STABILIZED ARCHES

by
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UNITED STATES ARMY
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A linear theory for the behavior of pressure-stabilized arches is derived and solutions to the resulting equations are obtained for uniform and concentrated loading for both simply supported and fixed-end restraints. The concentrated load solution is in the form of a Green's Function for application to more general loading situations. An experimental investigation of the behavior of pressure-stabilized arches is described, including measurement of fabric stiffness in addition to arch loading. Experimental determinations of the deformation and load-carrying capacity of several arches are correlated with predictions from the theory. These

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20. Abstract (cont'd)

correlations establish the usefulness of the theory was a design tool.

PREFACE

This work was carried out as a part of an In-House Laboratory Independent Research project entitled, "Investigation of the Strength and Stability Characteristics of Pressurized Rib Structures." This work was initiated in response to the findings of a systems analysis which identified the pressurized rib concept as being the most promising structural alternative for meeting the Army requirement for lightweight highly mobile tentage. As is indicated by the references cited in this report, two previous reports describing a similar investigation of pressure-stabilized beams have been published. In the reference citation the organizations "US Army Natick Laboratories" and "US Army Natick Development Center" refer to the organization now called the "US Army Natick Research and Development Command". The author wishes to express his thanks to Colette Klarman, formerly of the Data Analysis Office, for her effort in writing the computer program for carrying out the solution of the equations developed in this report and for plotting the theoretical and experimental results.

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INTRODUCTION

The use of pressure-stabilized structures for Army field shelters has been the subject of considerable attention over the past decade with emphasis on the single-wall and double-wall type of shelter, as indicated by references 1 and 2. A systems analysis of Army field shelter needs for the 1985 time frame revealed that the requirements for lightweight tentage of low package bulk and with minimum setup and disassembly times might be most effectively achieved with a tent having a frame of highly pressurized (compared with present air-supported tents) structural elements supporting a lightweight fabric barrier, as illustrated in Figure 1. As a result of the potential indicated by the systems analysis an investigation of the structural behavior of pressure-stabilized structural elements was begun with the objective being to develop a design capability which would permit the concept to be evaluated for use in Army tentage. The results of a theoretical and experimental study of the behavior of pressure-stabilized beams under load are presented in references 3 and 4. However, as is illustrated in Figure 1, pressure-stabilized arches will be used in this concept for tentage and thus design data for these structural elements are required. Such design data are presented in this report in the form of a theory for the deformation of pressure-stabilized arches together with experimental confirmation of the prediction made with this theory.

A review of previous work on the behavior of pressure-stabilized structural elements is given in references 1 and 2. The only additional work known to the author of interest relative to the behavior of arches is the theoretical study of the stability of pressure-stabilized rings given in reference 5. This work dealt entirely with the stability problem for the complete ring and these results were not used in the present work because it was believed that a more consistent theory would be obtained by adopting the ideas of superposition of small displacements on larger ones as was done for the pressure-stabilized beam problem in reference 3.

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1. Dietz, A. E., R. B. Proffitt, R. S. Chabot, E. L. Moak, and C. J. Monego; Wind Tunnel Tests and Analyses for Ground-Mounted, Air-Supported Structures (revised); US Army Natick Laboratories Technical Report 70–7–GP; 1969.

2. Dietz, A. E., R. B. Proffitt, R. S. Chabot, and E. L. Moak; Design Manual for Ground-Mounted Air-Supported Structures (Single-and Double-Wall) (revised); US Army Natick Laboratories Technical Report 69–59–GP; 1969.

3. Steeves, Earl C.; A Linear Analysis of the Deformation of Pressure Stabilized Beams; US Army Natick Laboratories Technical Report 75–47–AMEL; 1975.

4. Steeves, Earl C.; Behavior of Pressure Stabilized Beams Under Load; US Army Natick Development Center Technical Report 75–82–AMEL; 1975.

5. Weeks, G. E.; Buckling of a Pressurized Toroidal Ring Under Uniform External Loading; NASA TN D–4124; 1967.

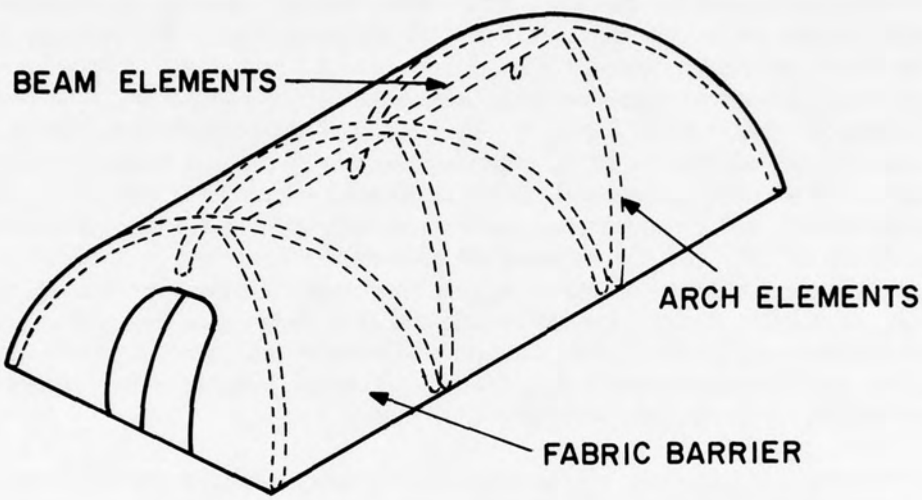


Figure 1. Tent Concept Using Pressure Stabilized Structural Elements

Included in this report is the derivation of the equations describing the behavior of pressure-stabilized arches under loads both in the plane of the arch and normal to the plane of the arch. The solution of the equations for the in-plane problem are presented along with a computer program to carry out this solution numerically. The experimental study is also descirbed, including the loading tests on arches and the measurement of the elastic properties of the fabric from which the arches were made. The results of this experimental study are compared with the predictions of the theory.

THEORETICAL ANALYSIS

This section of the report is concerned with development of the theory to predict the behavior of pressure-stabilized arches under load. We begin with a derivation of the governing equations for deformation in the plane of the arch and normal to the plane of the arch. These sets of equations are uncoupled and the solution to the in-plane set is obtained here and a computer program to carry out the solution numerically is described.

Derivation of Governing Equations

Fundamental Principles

We consider a surface in the form of a circular torus with a circular cross-section, as shown on Figure 2. Points on the surface are located by specification of the coordinates θ_1 and θ_2, and the differential arc element and radii of curvature on the surface are given by

$$ds^2 = [R + a\cos(\theta_1)]^2 d\theta_2^2 + a^2 d\theta_1^2 \tag{1a}$$

$$1/R_1 = 1/a \tag{1b}$$

$$1/R_2 = \cos(\theta_1)/[a\cos(\theta_1) + R] \tag{1c}$$

In equation (1), a is the cross-section radius and R is the radius of the torus centerline. Assuming that the membrane state of stress is present, the principle of minimum potential energy is written as

$$\delta \int_A (1/2)[N_{11}\epsilon_{11} + N_{22}\epsilon_{22} + 2N_{12}\epsilon_{12} - 2Pv_3]\,dA = 0 \tag{2}$$

Where N_{ij}, ϵ_{ij}, P, and V_3 are, respectively, the stress resultants, the surface strains, the inflation pressure, and the normal displacement. The surface strains can be expressed in terms of the displacements and their derivatives for the toroidal coordinates as:

$$\epsilon_{11} = (1/a)[v_{1,1} + v_3 + (1/2a)(-v_{3,1} + v_1)^2] \tag{2a}$$

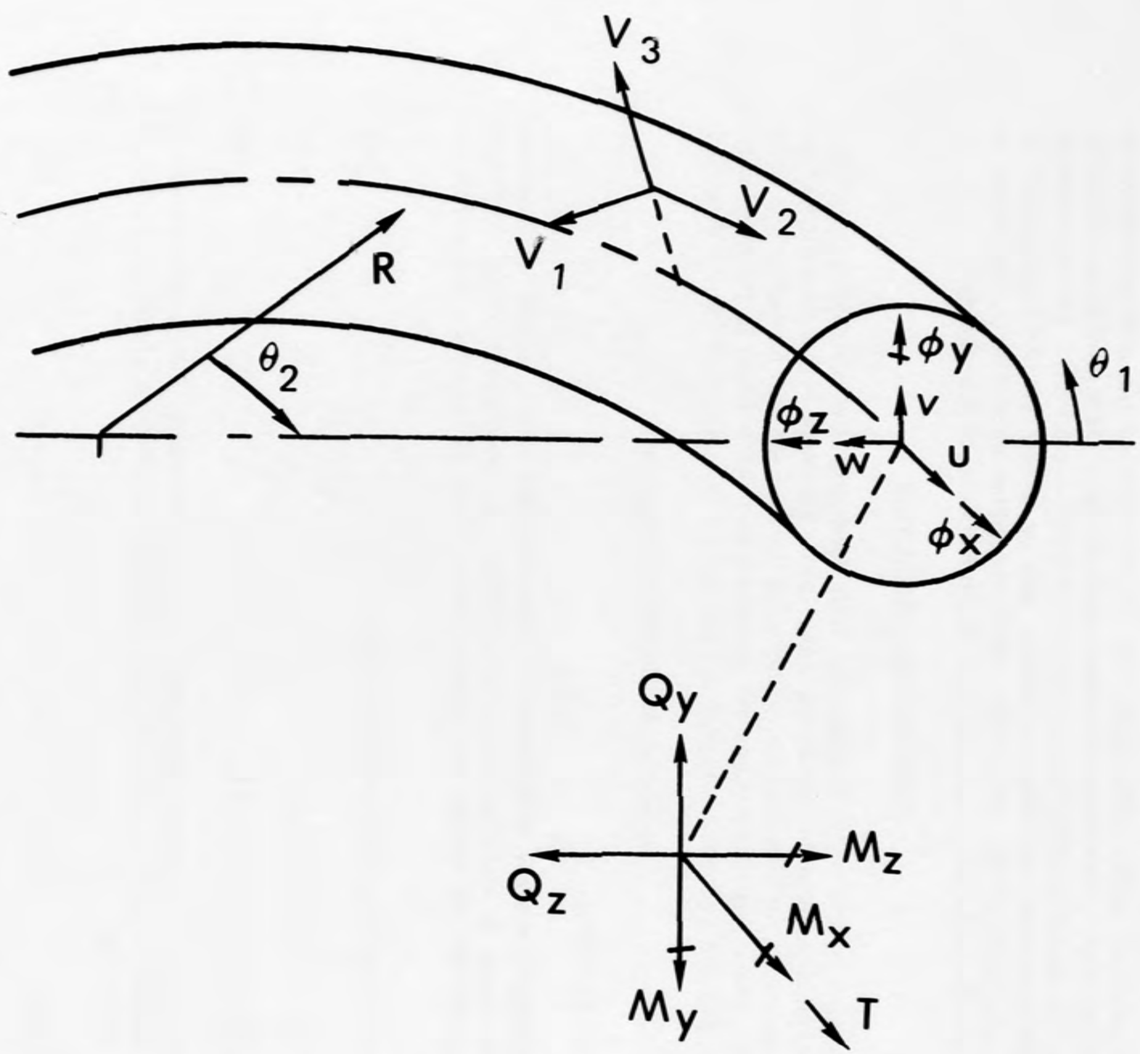


FIGURE 2. COORDINATES, DISPLACEMENT COMPONENTS AND INTERNAL FORCES

$$\epsilon_{22} = (1/\alpha_2)[v_{2,2} - v_1\sin(\theta_1) + v_3\cos(\theta_1) + (1/2\alpha_2)(-v_{3,2} + v_2\cos(\theta_1))^2] \tag{2b}$$

$$\epsilon_{12} = (1/2\alpha_2)[(R/a + \cos(\theta_1))v_{2,1} + v_{1,2} + v_2\sin(\theta_1) + (1/a)(-v_{3,1} + v_1)(-v_{3,2} + v_2\cos(\theta_1))] \tag{2c}$$

In equations (2) $\alpha_2 = (R + a\cos(\theta_1))$. To complete the basis for the theory we specify an orthotropic law relating the stress resultants and the surface strains as

$$N_{11} = C_{11}\epsilon_{11} + C_{12}\epsilon_{22} \tag{3a}$$

$$N_{22} = C_{12}\epsilon_{11} + C_{22}\epsilon_{22} \tag{3b}$$

$$N_{12} = 2C_{33}\epsilon_{12} \tag{3c}$$

It has been assumed in writing equations (3) that the lines of orthotropic symmetry are parallel to the coordinate lines.

Energy Principle for Deformation about Pressurized State

The derivation now proceeds identically with that for pressure-stabilized beams in reference 3 by assuming that the displacements v_i can be expressed as the sum of displacements w_i due to pressurization and u_i due to applied load as

$$v_i = w_i + u_i \tag{4}$$

Substitution of this displacement assumption into the strain-displacement relation (2) yields

$$\epsilon_{ij} = \overset{\circ}{\epsilon}_{ij} + \epsilon'_{ij} + \epsilon''_{ij} \tag{5}$$

$$i = 1,2$$
$$j = 1,2$$

These supplementary strain measures are defined in terms of the displacements w_i and u_i as

$$\overset{\circ}{\epsilon}_{11} = (1/a)[w_{1,1} + w_3 + (1/2a)(-w_{3,1} + w_1)^2]$$

$$\epsilon'_{11} = (1/a)[u_{1,1} + u_3 + (1/a)(-w_{3,1} + w_1)(-u_{3,1} + u_1)]$$

$$\epsilon''_{11} = (1/2a^2)(-u_{3,1} + u_1)^2 \tag{6}$$

$$\overset{\circ}{\epsilon}_{22} = (1/\alpha_2)[w_{2,2} - w_1\sin(\theta_1) + w_3\cos(\theta_1) + (1/2\alpha_2)(-w_{3,2} + w_2\cos(\theta_1))^2]$$

$$\epsilon'_{22} = (1/\alpha_2)[u_{2,2} - u_1\sin(\theta_1) + u_3\cos(\theta_1) + (1/\alpha_2)(-w_{3,2} + w_2\cos(\theta_1))(-u_{3,2} + u_2\cos(\theta_1))]$$

$$\epsilon''_{22} = (1/2\alpha_2^2)[-u_{3,2} + u_2\cos(\theta_1)]^2$$

$$\epsilon^{\circ}_{12} = (1/2\alpha_2)[(R/a + \cos(\theta_1))w_{2,1} + w_{1,2} + w_2\sin(\theta_1) + (1/a)(-w_{3,1} + w_1)(-w_{3,2} + w_2\cos(\theta_1))] \tag{6}$$

$$\epsilon'_{12} = (1/2\alpha_2)[(R/a + \cos(\theta_1))u_{2,1} + u_{1,2} + u_2\sin(\theta_1) + (1/a)(-w_{3,1} + w_1)(-u_{3,2} + u_2\cos(\theta_1)) + (1/a)(-u_{3,1} + u_1)(-w_{3,2} + w_2\cos(\theta_1))]$$

$$\epsilon''_{12} = (1/2a\alpha_2)[-u_{3,1} + u_1][-u_{3,2} + u_2\cos(\theta_1)]$$

The $\overset{\circ}{\epsilon}_{ij}$ are the complete nonlinear strain measures for the pressurized state expressed in terms of the displacements w_i. The ϵ'_{ij} are the linear strain measures in the displacements u_i due to applied load and also contain the terms coupling the pressurized state and that due to applied load. The ϵ''_{ij} are the nonlinear terms of the strains due to applied load and they can be interpreted as the products of the rotations of a surface line element about the i^{th} and j^{th} axes. Corresponding to these supplementary strain measures is a set of stress measures which represent the total stress components as

$$N_{ij} = \overset{\circ}{N}_{ij} + N'_{ij} + N''_{ij} \qquad i = 1,2 \quad j = 1,2 \tag{7}$$

Expressions for the stress measures $\overset{\circ}{N}_{ij}$, N'_{ij}, and N''_{ij} in terms of the strain measures $\overset{\circ}{\epsilon}_{ij}$, ϵ'_{ij}, and ϵ''_{ij} are found by substitution of equation (5) into the stress strain law (3) which yields

$$\begin{aligned}
\overset{\circ}{N}_{11} &= C_{11}\overset{\circ}{\epsilon}_{11} + C_{12}\overset{\circ}{\epsilon}_{22} \\
N'_{11} &= C_{11}\epsilon'_{11} + C_{12}\epsilon'_{22} \\
N''_{11} &= C_{11}\epsilon''_{11} + C_{12}\epsilon''_{22} \\
\overset{\circ}{N}_{22} &= C_{12}\overset{\circ}{\epsilon}_{11} + C_{22}\overset{\circ}{\epsilon}_{22} \\
N'_{22} &= C_{12}\epsilon'_{11} + C_{22}\epsilon'_{22} \\
N''_{22} &= C_{12}\epsilon''_{11} + C_{22}\epsilon''_{22}
\end{aligned} \tag{8}$$

$$
\begin{aligned}
\overset{\circ}{N}_{12} &= 2C_{33}\overset{\circ}{\epsilon}_{12} \\
N'_{12} &= 2C_{33}\epsilon'_{12} \\
N''_{12} &= 2C_{33}\epsilon''_{12}
\end{aligned} \tag{8}
$$

Substitution of equations (4), (5) and (7) into the energy principle (2) and retaining terms up to those quadratic in the displacement components u_i yields

$$
\begin{aligned}
\delta \int_A 1/2[(\overset{\circ}{\epsilon}_{11}\overset{\circ}{N}_{11} + \overset{\circ}{\epsilon}_{22}\overset{\circ}{N}_{22} + 2\overset{\circ}{\epsilon}_{12}\overset{\circ}{N}_{12} - 2Pw_3) + 2(\epsilon'_{11}\overset{\circ}{N}_{11} + \epsilon'_{22}\overset{\circ}{N}_{22} + \\
2\epsilon'_{12}\overset{\circ}{N}_{12} - Pu_3) + (\epsilon'_{11}N'_{11} + \epsilon'_{22}N'_{22} + 2\epsilon'_{12}N'_{12} + 2\epsilon''_{11}\overset{\circ}{N}_{11} + \\
2\epsilon''_{22}\overset{\circ}{N}_{22} + 4\epsilon''_{12}\overset{\circ}{N}_{12} - 2F_1u_1 - 2F_2u_2 - 2F_3u_3)]\,dA = 0
\end{aligned} \tag{9}
$$

In writing this relationship the work due to the applied forces F_i which do work only through the displacement components u_i (i = 1,3) has been added, and the following identities which can be proven through the stress-strain law have been used

$$
\begin{aligned}
\overset{\circ}{\epsilon}_{11}N'_{11} + \overset{\circ}{\epsilon}_{22}N'_{22} &= \epsilon'_{11}\overset{\circ}{N}_{11} + \epsilon'_{22}\overset{\circ}{N}_{22} \\
\overset{\circ}{\epsilon}_{12}N'_{12} &= \epsilon'_{12}\overset{\circ}{N}_{12} \\
\overset{\circ}{\epsilon}_{11}N''_{11} + \overset{\circ}{\epsilon}_{22}N''_{22} &= \epsilon''_{11}\overset{\circ}{N}_{11} + \epsilon''_{22}\overset{\circ}{N}_{22} \\
\overset{\circ}{\epsilon}_{12}N''_{12} &= \epsilon''_{12}\overset{\circ}{N}_{12}
\end{aligned} \tag{10}
$$

To yield the governing equation for deformation about the pressurized state, which is described by the displacement components u_i, the variation in equation (9) must be interpreted so as to be carried out with respect to the components u_i. Since the expression inclosed in the first set of parenthesis in equation (9) is independent of the components u_i, its variation must vanish. Carrying out the variation of the expression in the second set of parenthesis and performing the required integration by parts, one finds that the coefficients of the variational displacements δu_i are the equilibrium equations governing the pressurized state which must vanish if the stress resultants $\overset{\circ}{N}_{11}$, $\overset{\circ}{N}_{22}$, and $\overset{\circ}{N}_{12}$ represent an equilibrium state. Taking these facts into account, the following is the energy principle for deformation about the pressurized state:

$$
\begin{aligned}
\delta \int_A 1/2(\epsilon'_{11}N'_{11} + \epsilon'_{22}N'_{22} + 2\epsilon'_{12}N'_{12} + 2\epsilon''_{11}\overset{\circ}{N}_{11} + 2\epsilon''_{22}\overset{\circ}{N}_{22} + \\
4\epsilon''_{12}\overset{\circ}{N}_{12} - 2F_1u_1 - 2F_2u_2 - 2F_3u_3)\,dA = 0
\end{aligned} \tag{11}
$$

Since the stresses $\overset{\circ}{N}_{11}$, $\overset{\circ}{N}_{22}$, and $\overset{\circ}{N}_{12}$ due to pressurization are considered known quantities like the elastic constants, this energy expression is quadratic in the displacements u_i and will yield a set of linear equations. Thus the displacement assumption (4) has provided a means for linearizing the problem.

Simplified Displacement Approximation

As indicated, we have been able to linearize the problem while including the effect of pressurization but we still have a problem in two independent coordinates which will be described in terms of a set of linear partial differential equations. The present objective is to simplify the mathematical form by introducing a displacement approximation which specifies the form of the displacement components with respect to the meridional coordinate, θ_1. This displacement approximation describes the deformation in terms of rigid body motions of the cross-section; that is, the cross-section retains its shape during deformation. The approximation is the one used in beam theory. Here, however, transverse shear deformation is included. This approximation is stated mathematically by expressing the displacement components u_i in terms of the three displacements of the cross-section U, V, W and the rotations of the cross-section about three mutually perpendicular axes, ϕ_X, ϕ_Y, and ϕ_Z as:

$$u_1 = a\phi_X + W\sin(\theta_1) + V\cos(\theta_1) \tag{12a}$$

$$u_2 = U - \phi_Y a\cos(\theta_1) - \phi_Z a\sin(\theta_1) \tag{12b}$$

$$u_3 = V\sin(\theta_1) - W\cos(\theta_1) \tag{12c}$$

These cross-section displacements which are functions of θ_2 only are shown graphically in Figure 2. These displacement approximations can be used to formulate one-dimensional strain measures, stress resultants, and a one-dimensional energy principle.

The one-dimensional strain measures are obtained by substituting equations (12) into equations (6) yielding

$$\begin{aligned}
\epsilon'_{11} &= 0 \\
\epsilon''_{11} &= \phi_X^2/2 \qquad\qquad (13) \\
\epsilon'_{22} &= (1/\alpha_2)[U' - W) - \phi'_Y a\cos(\theta_1) - (\phi'_Z + \phi_X)a\sin(\theta_1)] \\
\epsilon''_{22} &= (1/2\alpha_2^2)[(U + W')\cos(\theta_1) - V'\sin(\theta_1) - \phi_Y a\cos^2(\theta_1) - \phi_Z a\sin(\theta_1)\cos(\theta_1)]^2 \\
\epsilon'_{12} &= (1/2\alpha_2)[a(\phi'_X - \phi_Z) + (W' + U + R\phi_Y)\sin(\theta_1) + (V' - R\phi_Z)\cos(\theta_1)] \\
\epsilon''_{12} &= (1/2\alpha_2)[\phi_X((U + W')\cos(\theta_1) - V'\sin(\theta_1) - \phi_Y a\cos^2(\theta_1) - \\
&\qquad \phi_Z a\sin(\theta_1)\cos(\theta_1))]
\end{aligned}$$

At this point we introduce two additional assumptions. The first of these assumes that the deformation is such that a uniaxial state of strain results which implies $\epsilon_{11} = 0$. This is consistent with the displacement approximation (12) to the first order and requires

that we neglect the second order term ϕ_X^2 and set $\epsilon_{11}'' = 0$. The second of these additional assumptions is that the shear strain can be adequately represented by a linear expression in the displacements and their derivatives. This assumption implies that $\epsilon_{12}'' = 0$. This assumption can be further justified by noting that in the energy expression (11) ϵ_{12}'' is a coefficient of $\overset{\circ}{N}_{12}$ which is the shear stress component for the pressurized state. Since this stress component vanishes for internal pressure loading, the nonlinear term being neglected in the shear strain would make no contribution to the energy.

The one-dimensional energy principle is obtained by incorporating the above assumptions in the energy principle (11) and rewriting it as a quadratic in the strain measures as:

$$\delta \int_A 1/2 \; [C_{22}(\epsilon_{22}')^2 + 4C_{33}(\epsilon_{12}')^2 + 2N_{22}^\circ \epsilon_{22}'' \\ -2(F_1 u_1 + F_2 u_2 + F_3 u_3)]\, dA = 0 \tag{14}$$

With the substitution of equations (12) and (13) into this expression and integrating with respect to θ_1 from 0 to 2π the one-dimensional energy principle is obtained as

$$\delta \int_{\theta_2} \pi/2 [\frac{C_{22}}{R^2}[2(U' - W)^2 + Z_2(\frac{a}{R})^2(U' - W + R\phi_Y')^2 + a^2 Z_3(\phi_Z' + \phi_X)^2] + \\ \frac{C_{33}}{R^2}[2a^2(\phi_X' - \phi_Z)^2 + Z_2(\frac{a}{R})^2(a(\phi_X' - \phi_Z) - \frac{R}{a}(V' - R\phi_Z))^2 + \\ Z_3(W' + U + R\phi_Y)^2] + \frac{N_{22}^\circ}{R^2}[(U + W')^2 + Z(\frac{a}{R})^2(U + W' + R\phi_Y)^2 + \\ (V')^2 + Z_4(\frac{a}{R})^2(V' - R\phi_Z)^2] - 2[Uf_2 + Wf_1 + Vf_3 + \\ a\phi_X f_4 - a\phi_Y f_5 - a\phi_Z f_6]]\, aR d\theta_2 = 0 \tag{15}$$

The details of this development are presented in Appendix A.

As derived in Appendix B, expressions for the one-dimensional stress resultants and moments in terms of the cross-section displacements and rotations are

$$t = \pi C_{22}\frac{a}{R}[(2 + (\frac{a}{R})^2 Z_2)(U' - W) + (\frac{a}{R})^2 Z_2 R\phi_Y'] \tag{16a}$$

$$q_Y = \pi C_{33}\frac{a}{R} Z_2[(V' - R\phi_Z) - R(\frac{a}{R})^2(\phi_X' - \phi_Z)] - \\ \pi N_{22}^\circ \frac{a}{R}[- Z_3 V' + (\frac{a}{R})^2 R Z_4 \phi_Z] \tag{16b}$$

$$q_z = \pi C_{33} Z_3 \frac{a}{R}[W' + U + R\phi_y] + \pi N^{\circ}_{22}\frac{a}{R}[Z_2(U + W') + (\frac{a}{R})^2 ZR\phi_y] \tag{16c}$$

$$m_x = \pi C_{33} a(\frac{a}{R})^2 [R(2 + (\frac{a}{R})^2 Z_2)(\phi_x' - \phi_z) - Z_2(V' - R\phi_z)] \tag{16d}$$

$$m_y = -\pi C_{22} Z_2 a(\frac{a}{R})^2 [U' - W + R\phi_y'] \tag{16e}$$

$$m_z = -\pi C_{22} Z_3 aR(\frac{a}{R})^2 (\phi_z' + \phi_x) \tag{16f}$$

The energy principle (15) can be expressed in terms of these stress resultants and moments by carrying out the variation and substituting equations (16) to give

$$\begin{aligned} &[\frac{1}{R} t\,\delta(U' - W) - \frac{1}{R} m_y \varphi_y' - \frac{1}{R} m_z \delta(\phi_z' + \phi_x) + \\ &\frac{1}{R} m_x \delta(\phi_x' - \phi_z) + \frac{1}{R} q_y \delta(V' - R\phi_z) + \frac{\pi}{R} N^{\circ}_{22} \frac{a}{R} V'\delta(R\phi_z) \\ &+ \frac{1}{R} q_z \delta(U + W' \;\; R\phi_y) - \frac{\pi}{R} N^{\circ}_{22} \frac{a}{R}(U + W')\delta(R\phi_y) \\ &- \pi a(f_2 \delta U + f_1 \delta W + f_3 \delta V + af_4 \delta\phi_x \\ &- af_5 \delta\phi_y - af_6 \delta\phi_z)]\, Rd\theta_2 = 0 \end{aligned} \tag{17}$$

Observation of the expression leads to the following definition of the one-dimensional strain measures

$$e = \frac{1}{R}(U' - W) \tag{18a}$$

$$\kappa_y = \frac{1}{R}\phi_y' \tag{18b}$$

$$\kappa_z = \frac{1}{R}(\phi_z' + \phi_x) \tag{18c}$$

$$\kappa_x = \frac{1}{R}(\phi_x' - \phi_z) \tag{18d}$$

$$\gamma_z = \frac{1}{R}(U + W' \;\; R\phi_y) \tag{18e}$$

$$\gamma_y = \frac{1}{R}(V' - R\phi_z) \tag{18f}$$

This completes the derivation of the one-dimensional theory. The governing equations in terms of the cross-section displacements and rotations can be obtained directly from equation (15) by application of the calculus of variations. Here the alternate procedure of obtaining the equilibrium equation in terms of the stress resultants and moments will be used. The governing equations in terms of the displacements and rotation are then obtained by use of the stress-displacement relations (16).

Governing Differential Equations

The equilibrium equations in terms of the stress resultants and moments are obtained from equation (17) by integrating by parts where possible to yield

$$
\begin{aligned}
\int_{-\alpha}^{\alpha} \Big\{ & [\frac{1}{R}(-t' + q_z) - \pi a f_2]\,\delta U - [\frac{1}{R}(t + q_z') + \pi a f_1]\,\delta W + \\
& [\frac{1}{R} m_y' + q_z - \frac{\pi a}{R} \overset{\circ}{N}_{22}(U + W') + \pi a^2 f_5]\,\delta\phi_y - [\frac{1}{R} q_y' + \pi a f_3]\,\delta V + \\
& [\frac{1}{R}(m_z' - m_x) - q_y + \frac{\pi a}{R} \overset{\circ}{N}_{22} V' + \pi a^2 f_6]\,\delta\phi_z - \\
& [\frac{1}{R}(m_z + m_x') + \pi a^2 f_4]\,\delta\phi_x \Big\} R d\theta_2 + \\
& (t\,\delta U - m_y \delta\phi_y - m_z \delta\phi_z + m_x \delta\phi_x + q_y \delta V + q_z \delta W]\Big|_{-\alpha}^{\alpha} = 0
\end{aligned}
\tag{19}
$$

Since the variational displacements are independent, satisfaction of the above equality requires that

$$\frac{1}{R}(-t' + q_z) - \pi a f_2 = 0 \tag{20a}$$

$$-\frac{1}{R}(t + q_z') - \pi a f_1 = 0 \tag{20b}$$

$$\frac{1}{R} m_y' + q_z - \pi \frac{a}{R} \overset{\circ}{N}_{22}(U + W') + \pi a^2 f_5 = 0 \tag{20c}$$

$$-\frac{1}{R} q_y' - \pi a f_3 = 0 \tag{20d}$$

$$\frac{1}{R}(m_z' - m_x) - q_y + \pi \frac{a}{R} \overset{\circ}{N}_{22} V' + \pi a^2 f_6 = 0 \tag{20e}$$

$$-\frac{1}{R}(m_z + m_x') - \pi a^2 f_4 = 0 \tag{20f}$$

over the region $-\alpha<\theta_2<\alpha$. On the boundaries $\theta_2=-\alpha$ and $\theta_2=\alpha$ one of each of the following pairs of parameters must be specified

$$\begin{array}{lll} t & \text{or} & U \\ m_y & \text{or} & \phi_y \\ q_z & \text{or} & W \\ m_z & \text{or} & \phi_z \\ m_x & \text{or} & \phi_x \\ q_y & \text{or} & V \end{array} \tag{21}$$

To complete the development, we substitute the stress-displacement relations (16) into the equilibrium equations to give the following governing equations for the displacements and rotations

$$\begin{aligned} &-C_{22}\frac{a}{R^2}\,(2+(\frac{a}{R})^2 Z_2)U'' + \frac{a}{R^2}\,(C_{33}Z_3 + N^{\circ}_{22}Z_2)U \\ &\quad -(\frac{a}{R})^3 C_{22}Z_2\phi_y'' + \frac{a}{R}(C_{33}Z_3 + (\frac{a}{R})^2 N^{\circ}_{22}Z)\phi_y \\ &\quad +\frac{a}{R^2}\,[C_{22}(2+(\frac{a}{R})^2 Z_2) + C_{33}Z_3 + N^{\circ}_{22}Z_2]W' - af_2 = 0 \end{aligned} \tag{22a}$$

$$\begin{aligned} &-\frac{a}{R^2}\,(C_{33}Z_3 + N^{\circ}_{22}Z_2)W'' + C_{22}\frac{a}{R^2}\,(2+(\frac{a}{R})^2 Z_2)W \\ &\quad -\frac{a}{R^2}\,[C_{22}(2+(\frac{a}{R^2})Z_2) + C_{33}Z_3 + N^{\circ}_{22}Z_2]U' \\ &\quad -\frac{a}{R}[(\frac{a}{R})^2 C_{22}Z_2 + C_{33}Z_3 + (\frac{a}{R})^2 N^{\circ}_{22}Z]\phi_y' - af_1 = 0 \end{aligned} \tag{22b}$$

$$\begin{aligned} &-(\frac{a}{R})^2 aC_{22}Z_2\phi_y'' + (C_{33}Z_3 + (\frac{a}{R})^2 N^{\circ}_{22}Z)a\phi_y \\ &\quad -(\frac{a}{R})^3 C_{22}Z_2U'' + \frac{a}{R}(C_{33}Z_3 + (\frac{a}{R})^2 N^{\circ}_{22}Z)U \\ &\quad +\frac{a}{R}[(\frac{a}{R})^2 C_{22}Z_2 + C_{33}Z_3 + (\frac{a}{R})^2 N^{\circ}_{22}Z]W' + a^2 f_5 = 0 \end{aligned} \tag{22c}$$

$$-\frac{a}{R^2}(C_{33}Z_2 + N^{\circ}_{22}Z_3)V'' + (\frac{a}{R})^3 C_{33}Z_2\phi_X''$$

$$+\frac{a}{R}[C_{33}Z_2(1 - (\frac{a}{R})^2) + (\frac{a}{R})^2 N^{\circ}_{22}Z_4]\phi_Z' - af_3 = 0 \qquad (22d)$$

$$-(\frac{a}{R})^2 aC_{22}Z_3\phi_Z'' + [C_{33}(2(\frac{a}{R})^2 + (1 - (\frac{a}{R})^2)^2 Z_2) + (\frac{a}{R})^2 N^{\circ}_{22}Z_4]a\phi_Z$$

$$-[(\frac{a}{R})^2 C_{22}Z_3 + C_{33}(\frac{a}{R})^2(2 - Z_2 + (\frac{a}{R})^2 Z_2)]a\phi_X' \qquad (22e)$$

$$-\frac{a}{R}[C_{33}Z_2(1 - (\frac{a}{R})^2) - N^{\circ}_{22}(1 - Z_3)]V' + a^2 f_6 = 0$$

$$-(\frac{a}{R})^2 aC_{33}(2 + (\frac{a}{R})^2 Z_2)\phi_X'' + (\frac{a}{R})^2 aC_{22}Z_3\phi_X$$

$$+(\frac{a}{R})^2[C_{22}Z_3 + C_{33}(2 - Z_2 + (\frac{a}{R})^2 Z_2)]a\phi_Z' \qquad (22f)$$

$$+ (\frac{a}{R})^3 C_{33}Z_2V'' - a^2 f_4 = 0$$

This gives a set of six linear second order differential equations with constant coefficients in the six cross-section displacement parameters. The first three of these equations describing the deformation in the plane of the arch are uncoupled from the last three which describe the deformation out of the plane of the arch. This completes the development of the theory for the structural behavior of pressure-stabilized arches. The theory is put in nondimensional form in Appendix C. Although the motivation for this derivation was to establish equations to describe or predict the behavior of pressure-stabilized arches under applied static loads, these equations also represent a theory for the elastic stability of arches. That is, if the applied loads, f_i, i=1, 6, are set equal to zero and homogenous boundary conditions are used, we have a set of homogenous equations for the determination of the values of N°_{22} giving nontrivial solution functions for the displacement parameters. These values of N°_{22} are the buckling stresses, and the solution functions are buckling mode shapes.

Discontinuity Conditions for Green's Function:

One additional item that will be required is a derivation of the discontinuity conditions required for obtaining a Green's function solution. The Green's function for structural problems can be interpreted as the deformation at a general point θ_2 due to a unit load applied at $\bar{\theta}_2$ The discontinuity conditions involve this unit load and internal stress resultants and moments. To derive these conditions we consider an arch having an angular span of 2α with the origin of the coordinate θ_2 positioned at the center of the span

as shown in Figure 3. The load applied at $\theta_2 = \bar{\theta}_2$ has some distribution with respect to the coordinate θ_1 as shown in Figure 3 and requires three components, $G_1(\theta_1)$, $G_2(\theta_1)$, and $G_3(\theta_1)$, for complete specification. The arch is divided into two regions, one on either side of the applied load or mathematically $-\alpha \leqslant \theta_2 < \bar{\theta}_2$ and $\bar{\theta}_2 < \theta_2 \leqslant \alpha$. To obtain the needed conditions the energy principle (11) is rewritten integrating over each of the regions separately and including the potential energy of the forces applied at $\theta_2 = \bar{\theta}_2$

$$\delta \int_{-\alpha}^{\bar{\theta}_2} \int_{0}^{2\pi} E(\theta_1, \theta_2)(1 + a/R\cos(\theta_1))Rad\theta_1 d\theta_2 +$$

$$\delta \int_{\bar{\theta}_2}^{\alpha} \int_{0}^{2\pi} E(\theta_1, \theta_2)(1 + a/R\cos(\theta_1))Rad\theta_1 d\theta_2 - \tag{23}$$

$$\delta \int_{0}^{2\pi} [G_1(\theta_1) u_1(\theta_1, \bar{\theta}_2) + G_2(\theta_1) u_2(\theta_1, \bar{\theta}_2) + G_3(\theta_1) u_3(\theta_1, \bar{\theta}_2)] ad\theta_1 = 0$$

where $E(\theta_1, \theta_2)$ is the energy density function and the displacement components in the third integral are evaluated at $\theta_2 = \bar{\theta}_2$. The simplified displacement approximations are substituted as done previously and the integration with respect to θ_1 carried out to give

$$\delta \int_{-\alpha}^{\bar{\theta}_2} E(\theta_2)aRd\theta_2 + \delta \int_{\bar{\theta}_2}^{\alpha} E(\theta_2)aRd\theta_2 -$$

$$\delta \Big\{ \pi a [g_2 U(\bar{\theta}_2) + g_1 W(\bar{\theta}_2) + g_3 V(\bar{\theta}_2) + \tag{24}$$

$$g_4 a\theta_x(\bar{\theta}_2) - g_5 a\phi_y(\bar{\theta}_2) - g_6 a\phi_z(\bar{\theta}_2)] \Big\} = 0$$

where

$$g_1 = \frac{1}{\pi} \int_0^{2\pi} (G_1(\theta_1)\sin(\theta_1) - G_3(\theta_1)\cos(\theta_1))d\theta_1 \tag{25a}$$

$$g_1 = \frac{1}{\pi} \int_0^{2\pi} G_2(\theta_1)d\theta_1 \tag{25b}$$

$$g_3 = \frac{1}{\pi} \int_0^{2\pi} (G_1(\theta_1)\cos(\theta_1) + G_3(\theta_1)\sin(\theta_1))d\theta_1 \tag{25c}$$

$$g_4 = \frac{1}{\pi} \int_0^{2\pi} G_1(\theta_1)d\theta_1 \tag{25d}$$

$$g_5 = \frac{1}{\pi} \int_0^{2\pi} G_2(\theta_1)\cos(\theta_1)d\theta_1 \tag{25e}$$

$$g_6 = \frac{1}{\pi} \int_0^{2\pi} G_2(\theta_1)\sin(\theta_1)d\theta_1 \tag{25f}$$

Over the region $-\alpha \leqslant \theta_2 \leqslant \bar{\theta}_2$, the energy density is expressed in terms of the displacement parameters U_1, W_1, V_1, ϕ_{x1}, ϕ_{y1}, and ϕ_{z1} and over the region $\bar{\theta}_2 < \theta_2 \leqslant \alpha$ in terms of U_2, W_2, V_2, ϕ_{x2}, ϕ_{y2} and ϕ_{z2}. If the variation in (24) is carried out, the result expressed in terms of stress resultants and moments, and the integration by parts performed, a set of six governing equations identical in form to (22) are obtained from each of the two regions being considered. What is of interest here are the boundary terms and the applied force at $\bar{\theta}_2$. This boundary term which must vanish is

$$
\begin{aligned}
& \left. t_1 \delta U_1 - m_{y1} \delta\phi_{y1} - m_{z1} \delta\phi_{z1} + m_{x1} \delta\phi_{x1} + q_{y1} \delta V_1 + q_{z1} \delta W_1 \right|_{-\alpha}^{\bar{\theta}_2} + \\
& \left. t_2 \delta U_2 - m_{y2} \delta\phi_{y2} - m_{z2} \delta\phi_{z2} + m_{x2} \delta\phi_{x2} + q_{y2} \delta V_2 + q_{z2} \delta W_2 \right|_{\bar{\theta}_2}^{\alpha} \\
& -a\pi[g_2 \delta U_1(\bar{\theta}_2) + g_1 \delta W_1(\bar{\theta}_2) + g_3 \delta V_1(\bar{\theta}_2) + g_4 a \delta\phi_{x1}(\bar{\theta}_2) \\
& \qquad -g_5 a \delta\phi_{y1}(\bar{\theta}_2) - g_6 a \delta\phi_{z1}(\bar{\theta}_2)] = 0
\end{aligned}
\tag{26}
$$

In writing (26) subscript 1 refers to a variable defined on the region $-\alpha \leqslant \theta_2 \leqslant \bar{\theta}_2$ and subscript 2 to a variable defined on $\bar{\theta}_2 < \theta_2 \leqslant \alpha$. The boundary terms at $\theta_2 = \pm\alpha$ give the boundary conditions at the ends of the arch which are, not unexpectedly, identical with those derived previously. The remaining boundary terms will yield the conditions that must hold at the applied load, but it is first necessary to state the conditions of continuity of deformation between the two regions. These conditions are simply

$$
\begin{aligned}
U_1(\bar{\theta}_2) &= U_2(\bar{\theta}_2) \\
W_1(\bar{\theta}_2) &= W_2(\bar{\theta}_2) \\
V_1(\bar{\theta}_2) &= V_2(\bar{\theta}_2) \\
\phi_{x1}(\bar{\theta}_2) &= \phi_{x2}(\bar{\theta}_2) \\
\phi_{y1}(\bar{\theta}_2) &= \phi_{y2}(\bar{\theta}_2) \\
\phi_{z1}(\bar{\theta}_2) &= \phi_{z2}(\bar{\theta}_2)
\end{aligned}
\tag{27}
$$

Thus the variations of these parameters must be equal and the boundary terms at $\overline{\theta}_2$ become

$$[t_1 - t_2 - a\pi g_2]\,\delta U_1(\overline{\theta}_2) + [-m_{y_1} + m_{y_2} + \pi a^2 g_5]\,\delta\phi_{y_1}(\overline{\theta}_2)$$

$$[q_{z1} - q_{z2} - \pi a g_1]\,\delta W_1(\overline{\theta}_2) + [-m_{z1} + m_{z2} + \pi a^2 g_6]\,\delta\phi_{z1}(\overline{\theta}_2) \tag{28}$$

$$[m_{x1} - m_{x2} - \pi a^2 g_4]\,\delta\phi_{x1}(\overline{\theta}_2) + [q_{y_1} - q_{y_2} - \pi a g_3]\,\delta V_1(\overline{\theta}_2) = 0$$

Each of the terms must vanish independently giving the discontinuity conditions as

$$t_1(\overline{\theta}_2) - t_2(\overline{\theta}_2) = \pi a g_2 \tag{29a}$$

$$m_{y1}(\overline{\theta}_2) - m_{y_2}(\overline{\theta}_2) = \pi a^2 g_5 \tag{29b}$$

$$q_{z1}(\overline{\theta}_2) - q_{z2}(\overline{\theta}_2) = \pi a g_1 \tag{29c}$$

$$m_{z1}(\overline{\theta}_2) - m_{z2}(\overline{\theta}_2) = \pi a^2 g_6 \tag{29d}$$

$$m_{x1}(\overline{\theta}_2) - m_{x2}(\overline{\theta}_2) = \pi a^2 g_4 \tag{29e}$$

$$q_{y_1}(\overline{\theta}_2) - q_{y_2}(\overline{\theta}_2) = \pi a g_3 \tag{29f}$$

These are the required conditions and they are expressed in nondimensional form in Appendix C.

Solution of the Governing Equations

Having derived a set of governing equations in the previous section we turn now to the solution of these equations. In what follows only the inplane deformation will be considered with solutions obtained in the form of a Green's Function and for an arch with a uniform normal load. The Green's Function provides a means for obtaining solutions for quite general loading by integration.

Homogenous Solution

The equations to be solved are (22a), (22b), and (22c) which are given in nondimensional form in Appendix C as (C22), (C23) and (C24). These are rewritten here in more compact form to aid in writing the solutions.

$$-H_1U'' + H_2U - H_3\phi_y'' + H_4\phi_y + (H_1 + H_2)W' = 0 \tag{30a}$$

$$-H_2W'' + H_1W - (H_3 + H_4)\phi_y' - (H_1 + H_2)U' = 0 \tag{30b}$$

$$-\rho H_3\phi_y'' + \rho H_4\phi_y - H_3U'' + H_4U + (H_3 + H_4)W' = 0 \tag{30c}$$

where

$$H_1 = 2d/\rho + dZ_2/\rho^3 \tag{31a}$$

$$H_2 = cZ_3/\rho + nZ_2/\rho \tag{31b}$$

$$H_3 = dZ_2/\rho^2 \tag{31c}$$

$$H_4 = cZ_3 + nZ/\rho^2 \tag{31d}$$

Since the homogenous solution is sought, the force parameters are set equal to zero. The equations have constant coefficients, so the solution is

$$U = Ae^{\omega\theta_2} \tag{32a}$$

$$\phi_y = Be^{\omega\theta_2} \tag{32b}$$

$$W = Ce^{\omega\theta_2} \tag{32c}$$

Substitution of this solution into (30) yields the following set of homogenous equations for A, B, and C:

$$\begin{bmatrix} H_2 - H_1\omega^2 & H_4 - H_3\omega^2 & (H_1 + H_2)\omega \\ H_4 - H_3\omega^2 & \rho(H_4 - H_3\omega^2) & (H_3 + H_4)\omega \\ -(H_1 + H_2)\omega & -(H_3 + H_4)\omega & H_1 - H_2\omega^2 \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = 0 \tag{33}$$

Existence of a nontrivial solution requires that ω be a solution of

$$\begin{aligned} &H_1H_4(H_4 - \rho H_2) + \omega^2[H_2H_3(\rho H_1 - H_3) + 2H_1H_4(H_4 - \rho H_2)] + \\ &\omega^4[H_1H_4(H_4 - \rho H_2) + 2H_2H_3(\rho H_1 - H_3)] + \\ &\omega^6[H_2H_3(\rho H_1 - H_3)] = 0 \end{aligned} \tag{34}$$

This is a sixth order equation in ω or a cubic equation in ω^2 which can be written in factored form as

$$(\omega^2 + 1)(\omega^2 + 1)[H_2H_3(\rho H_1 - H_3)\omega^2 + H_1H_4(H_4 - \rho H_2)] = 0 \tag{35}$$

for which the roots are $(-i, -i, i, i, \lambda, -\lambda)$ where

$$\lambda^2 = \frac{H_1 H_4 (\rho H_2 - H_4)}{H_2 H_3 (\rho H_1 - H_3)} = \frac{n\rho^2 (2 + Z_2/\rho^2)(cZ_3 + nZ/\rho^2)}{2Z_2 d (cZ_3 + nZ_2)} \tag{36}$$

Thus λ^2 is always positive and the homogenous solution, taking account of the repeated roots, is

$$U = A_1 \cosh(\lambda\theta_2) + A_2 \sinh(\lambda\theta_2) + A_3 \sin(\theta_2) + A_4 \cos(\theta_2) + A_5 \theta_2 \sin(\theta_2) + A_6 \theta_2 \cos(\theta_2) \tag{37a}$$

$$\phi_y = B_1 \cosh(\lambda\theta_2) + B_2 \sinh(\lambda\theta_2) + B_3 \sin(\theta_2) + B_4 \cos(\theta_2) + B_5 \theta_2 \sin(\theta_2) + B_6 \theta_2 \cos(\theta_2) \tag{37b}$$

$$W = C_1 \cosh(\lambda\theta_2) + C_2 \sinh(\lambda\theta_2) + C_3 \sin(\theta_2) + C_4 \cos(\theta_2) + C_5 \theta_2 \sin(\theta_2) + C_6 \theta_2 \cos(\theta_2) \tag{37c}$$

The satisfaction of (27) implies a relationship among the constants A_i, B_i, and C_i such that two sets can be expressed in terms of the third. Here A_i and B_i are expressed in terms of C_i as

$$\begin{aligned} A_1 &= \beta_1 C_2 \\ A_2 &= \beta_1 C_1 \\ A_3 &= \beta_3 C_4 + \beta_5 C_5 \\ A_4 &= -\beta_3 C_3 + \beta_5 C_6 \\ A_5 &= \beta_3 C_6 \\ A_6 &= -\beta_3 C_5 \end{aligned} \tag{38}$$

$$
\begin{aligned}
B_1 &= \beta_2 C_2 \\
B_2 &= \beta_2 C_1 \\
B_3 &= \beta_4 C_4 + \beta_6 C_5 \\
B_4 &= -\beta_4 C_3 + \beta_6 C_6 \\
B_5 &= \beta_4 C_6 \\
B_6 &= -\beta_4 C_5
\end{aligned}
\tag{39}
$$

Expressions for the determination of the β_i are given in Appendix D. With these relations among the constants the solution is

$$
\begin{aligned}
U = {} & C_1\beta_1 \sinh(\lambda\theta_2) + C_2\beta_1\cosh(\lambda\theta_2) - C_3\beta_3\cos(\theta_2) + C_4\beta_3\sin(\theta_2) + \\
& C_5(\beta_5\sin(\theta_2) - \beta_3\theta_2\cos(\theta_2)) + C_6(\beta_5\cos(\theta_2) + \beta_3\theta_2\sin(\theta_2))
\end{aligned}
\tag{40a}
$$

$$
\begin{aligned}
\phi_y = {} & C_1\beta_2 \sinh(\lambda\theta_2) + C_2\beta_2\cosh(\lambda\theta_2) - C_3\beta_4\cos(\theta_2) + C_4\beta_4\sin(\theta_2) + \\
& C_5(\beta_6\sin(\theta_2) - \beta_4\theta_2\cos(\theta_2)) + C_6(\beta_6\cos(\theta_2) + \beta_4\theta_2\sin(\theta_2))
\end{aligned}
\tag{40b}
$$

$$
\begin{aligned}
W = {} & C_1\cosh(\lambda\theta_2) + C_2\sinh(\lambda\theta_2) + C_3\sin(\theta_2) + C_4\cos(\theta_2) \\
& C_5\theta_2\sin(\theta_2) + C_6\theta_2\cos(\theta_2)
\end{aligned}
\tag{40c}
$$

This is the homogenous solution containing six unknown constants to be determined by the boundary conditions for a given set of applied loads.

Green's Function

Instead of attempting to find particular solutions for a variety of loading situations, a Green's function is obtained which will provide a solution for general load through integration. As stated above, the Green's Function has for structural problems the interpretation of the deflection at a general point θ_2 due to a unit load applied at $\overline{\theta}_2$. As in the derivation of the discontinuity conditions we consider a beam having an angular span of 2α with the origin located at midspan, as shown in Figure 3. Over the region $-\alpha \leqslant \theta_2 \leqslant \overline{\theta}_2$ a solution U_1, ϕ_{y_1}, and W_1 is defined in terms of integration constants C_i, $i = 1,6$ and over the region $\overline{\theta}_2 < \theta_2 \leqslant \alpha$ a solution U_2, ϕ_{y_2}, and W_2 is defined in terms of integration constants C_i, $i = 7,12$. Thus the complete solution is defined by 12 constants and 12 conditions must be specified for their determination. Six of these come from the boundary conditions, three on each end. The other six come from the three conditions of continuity of displacements and the three discontinuity conditions on the internal stress resultants and moments. If both ends are simply supported these conditions are

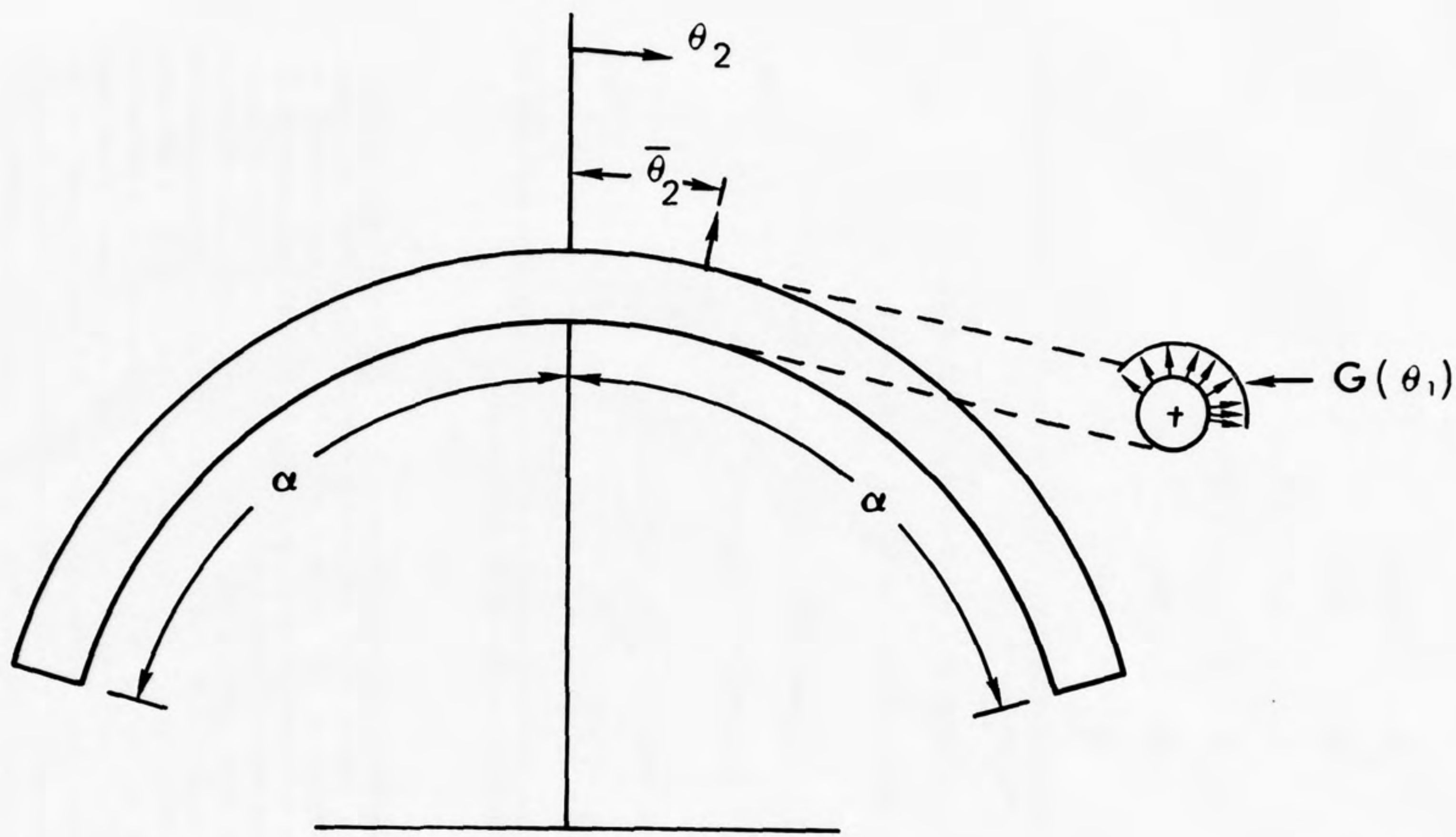


FIGURE 3. ILLUSTRATION OF THE LINE LOAD USED IN OBTAINING THE GREEN'S FUNCTION

$$
\begin{aligned}
&U_1(-\alpha) = 0 \\
&M_{y_1}(-\alpha) = 0 \\
&W_1(-\alpha) = 0 \\
&U_2(\alpha) = 0 \\
&M_{y_2}(\alpha) = 0 \\
&W_2(\alpha) = 0 \\
&U_1(\overline{\theta}_2) - U_2(\overline{\theta}_2) = 0 \\
&\phi_{y_1}(\overline{\theta}_2) - \phi_{y_2}(\overline{\theta}_2) = 0 \\
&W_1(\overline{\theta}_2) - W_2(\overline{\theta}_2) = 0 \\
&T_1(\overline{\theta}_2) - T_2(\overline{\theta}_2) = \pi\overline{g}_2 \\
&M_{y_1}(\overline{\theta}_2) - M_{y_2}(\overline{\theta}_2) = \pi\overline{g}_5 \\
&Q_{z_1}(\overline{\theta}_2) - Q_{z_2}(\overline{\theta}_2) = -\pi\overline{g}_1
\end{aligned}
\tag{41}
$$

and if both ends are fixed, the conditions $M_{y_1}(-\alpha) = 0$ and $M_{y_2}(\alpha) = 0$ are respectively replaced by $\phi_{y_1}(-\alpha) = 0$ and $\phi_{y_2}(\alpha) = 0$. Other boundary conditions at $\pm\alpha$ are also possible. These conditions give a system of 12 linear equations for the constants C_i, $i = 1,12$. The expressions for the coefficients of this system of equations are given in Appendix E.

Because only inplane deformation of the arch is being considered, the applied forces G_1, G_2, and G_3 must be symmetric about the plane of the arch. Two loading situations are of particular interest, normal load and vertical load. For normal loading, the components G_1 and G_2 vanish, therefore the nondimensional generalized forces $\overline{g}_2$ and $\overline{g}_5$ vanish and the load is described in terms of $\overline{g}_1$, and for a unit load $\overline{g}_1$ is set equal to unity. A vertical load is defined to be parallel to the line $\theta_2 = 0$. For a vertical load distribution $G(\theta_1)$ located at $\overline{\theta}_2$, the force components are

$$
\begin{aligned}
G_1 &= -G(\theta_1)\sin(\theta_1)\cos(\overline{\theta}_2) \\
G_2 &= -G(\theta_1)\sin(\overline{\theta}_2) \\
G_3 &= G(\theta_1)\cos(\theta_1)\cos(\overline{\theta}_2)
\end{aligned}
\tag{42}
$$

and the generalized forces are found by carrying out the integrals in the definition (25). For example, if we have two concentrated forces of magnitude G/2 located at $\theta_1 = \pm \hat{\theta}_1$ then

$$G(\theta_1) = (G/2)[\delta(\theta_1 + \hat{\theta}_1) + \delta(\theta_1 - \hat{\theta}_1)] \tag{43}$$

where δ here denotes the Dirac delta function. Substituting (43) and (42) into (25) and carrying out the integrals using the integral property of the delta function yields

$$\begin{aligned}
\bar{g}_1 &= -g\cos(\bar{\theta}_2) \\
\bar{g}_2 &= -g\sin(\bar{\theta}_2) \\
\bar{g}_3 &= -g\sin(\bar{\theta}_2)\cos(\hat{\theta}_1)
\end{aligned}$$

for a unit load, $g = G/\pi C_{22}$ is set equal to unity.

Uniformly Distributed Normal Load

The solution for a uniform normal load could be obtained by integration of the Green's function derived in the previous section. However, since a particular solution of the nonhomogenous equations for this loading condition is easily found it was decided to give the solution explicitly as the sum of this particular solution and the homogenous solution. For this loading condition we deal with the forces F_1, F_2, and F_3, with F_1 and F_2 vanishing. F_3 is independent of θ_2 and has a variation in θ_1 which is symmetrical about $\theta_1 = 0$ and is either constant or zero to give a distribution similar to that shown in Figure 4. Such a distribution will cause all the nondimensional force parameters to vanish except F_1 which will be a constant. Thus we need a particular solution of 30a, 30b, and 30c with F_1 appearing as a constant nonhomogenous term in (30b). The required particular solution is

$$\begin{aligned}
U &= 0 \\
\phi_Y &= 0 \\
W &= \bar{f}_1/H_1
\end{aligned} \tag{44}$$

The complete solution is the sum of this particular solution and the symmetric part of the homogenous solution

$$U = C_1\beta_1\sinh(\lambda\theta_2) + C_4\beta_3\sin(\theta_2) + C_5(\beta_5\sin(\theta_2) - \beta_3\theta_2\cos(\theta_2)) \tag{45a}$$

$$\phi_Y = C_1\beta_2\sinh(\lambda\theta_2) + C_4\beta_4\sin(\theta_2) + C_5(\beta_6\sin(\theta_2) - \beta_4\theta_2\cos(\theta_2)) \tag{45b}$$

$$W = C_1\cosh(\lambda\theta_2) + C_4\cos(\theta_2) + C_5\theta_2\sin(\theta_2) + \rho f_1/d(2 + Z_2/\rho^2) \tag{45c}$$

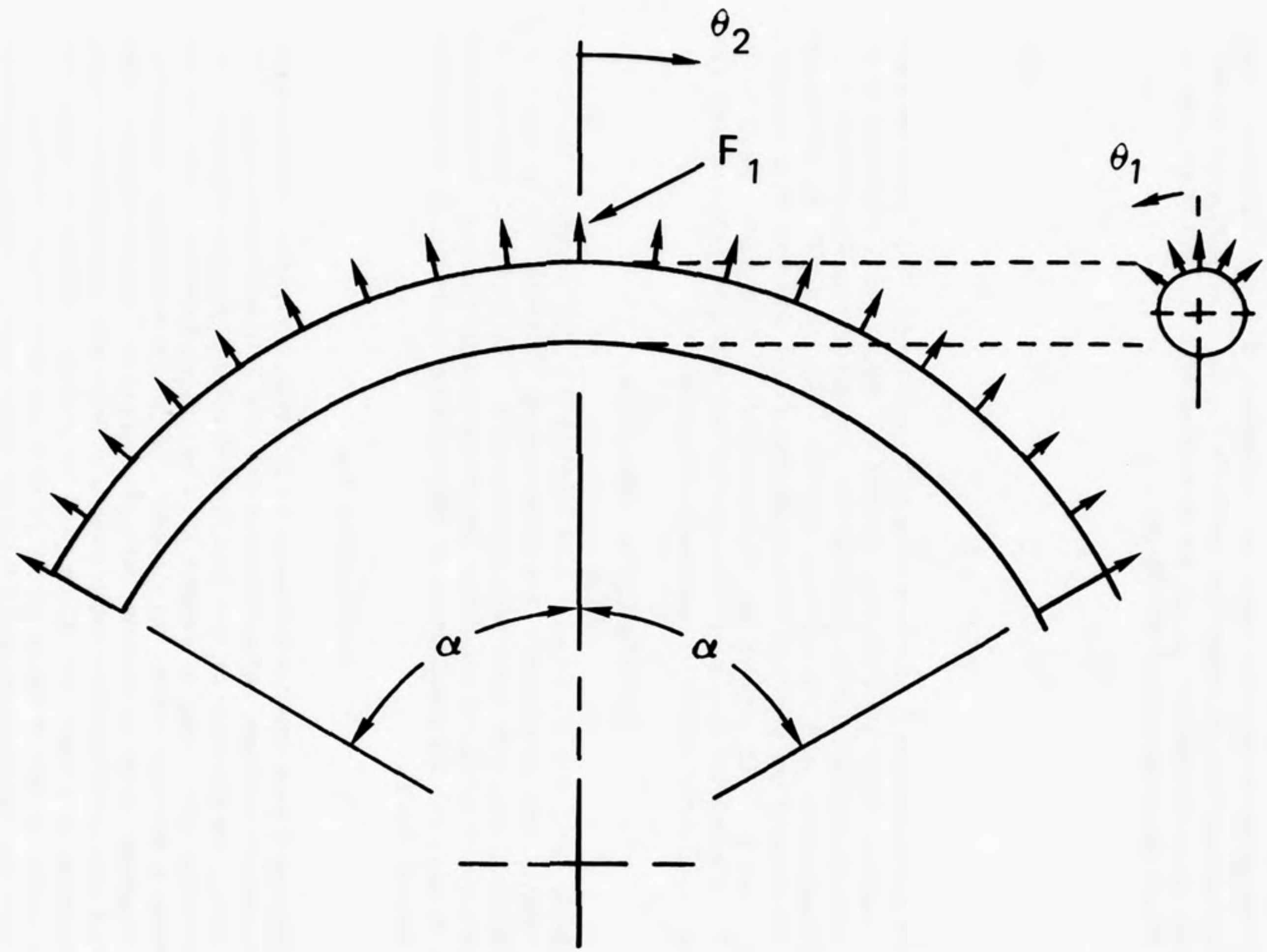


FIGURE 4. ILLUSTRATION OF UNIFORM NORMAL LOAD DISTRIBUTION

Only the symmetric part of the solution is included because the loading is symmetric about $\theta_2 = 0$, thus the deformation will be symmetric. Here symmetric deformation is used to characterize the situation where the displacement W is symmetric and U and ϕ_y are antisymmetric, thus the integration constants C_2, C_3, C_6 are set equal to zero. The remaining three constants C_1, C_4, C_5 are determined from the boundary conditions which for simply supported end conditions are

$$
\begin{aligned}
U(\alpha) &= 0 \\
M_y(\alpha) &= 0 \\
W(\alpha) &= 0
\end{aligned}
\tag{46}
$$

and for fixed end conditions $M_y(\alpha) = 0$ is replaced by $\phi_y(\alpha) = 0$. In both of these cases symmetric boundary conditions have been assumed. If the boundary conditions to be applied to each of the two ends are different, the complete homogenous solution must be used in conjunction with the particular solution. Equations (46) for the simply supported arch or with the substitution for fixed ends gives a system of three equations for C_1, C_4, and C_5. Expressions for the coefficients of this system are given in Appendix E. A computer program to carry out the computations associated with this solution and the Green's function is described in Appendix F.

EXPERIMENTAL ANALYSIS

To provide some data on which to base a judgment of the validity of the theory developed above, the deformation and load-carrying capability of a series of pressure-stabilized arches were determined experimentally. The elastic and shear moduli of the fabric used in making the arches were also measured experimentally. These moduli are needed to carry out the computations of the theoretical predictions for comparison with experimental results.

Arch Loading Tests

The objective of these tests was to measure the deformation behavior and load-carrying capacity of pressure stabilized arches of several sizes and inflation pressure levels. The flexibility, that is, the deflection per unit load, is taken as the measure of the deformation, and the wrinkling load is used to measure the load-carrying capability. The wrinkling load is defined as the load at which the maximum compressive stress due to applied load is equal in magnitude to the tensile stress due to pressurization. Thus any further increase in load would cause wrinkling of the fabric because it cannot support compressive stresses. The arches consist of a fabric skin which gives the structural strength to support the pressure, a bladder to prevent leakage of the inflation gas and end caps. The fabric skin was woven with beta fiberglass yarn using a three-dimensional weaving technique that results in a fabric having the natural contour of a torus. That is, when the arch is inflated it has the shape of a torus and no forces are required to maintain it in that shape. This

weaving process, which is described in references 6 and 7, produces a fabric in which the circumferential or fill yarn count is higher at the inner radius of the arch than at the outer radius. This nonuniformity of yarn count and the resulting natural contour or shape causes some difficulty in the determination of the fabric stiffness properties as will be discussed subsequently. The fabric used was a plain weave with an average yarn count in both warp and fill of 512/m using 133 Tex fiberglass yarns. Arches having cross-section radii of 0.038, 0.051, 0.076, and 0.089 m, all having an inner arch radius of 0.914 m, were woven and tested. The arch radius to cross-section radius ratios for these arches are then respectively 25, 19, 13, and 11.3.

A polyethylene film bladder was used to provide retention of the inflation gas. These bladders, made from 4-mil-thick film, were shaped to conform to the arch shape by cutting two segments of a circular annulus and joining them together by heat-sealing at the inner and outer radii. These bladders were made with the cross-section about 10% larger than that of the corresponding fabric. In doing this the strength requirements of the bladder were not a consideration. However, because of the oversizing of the bladders it was possible for folds to form in them, and when pressurized, the folds were flattened out resulting in cracks that caused leakage. This is believed to be a baldder material problem that could be cured by choosing a material with greater crack resistance.

The ends of the arches were closed and restrained with end cap assemblies shown in Figure 5. A schematic of this assembly is shown in Figure 6. As is shown, the fiberglass fabric and polyethylene bladder go through the inside of and wrap around the sealing ring. This arch-sealing ring combination is seated in the end cap on the rubber gasket. The arch and sealing ring are captured by the retaining ring which is bolted to the end cap to hold the assembly together and apply pressure to seal the arch for inflation. Inflation is accomplished through the port shown in Figure 6. As is seen in Figure 5, the end cap assembly was bolted to a steel plate so that it could not rotate. Thus the end restraint simulated fixed or clamped ends. Each arch had two end caps placed so that the arches had an angular span of π radians.

6. Koppelman, Edward and Arthur R. Campman; Woven Fabrics and Method of Weaving; US Patent No. 2998030; Aug 29, 1961.

7. Koppelman, Edward and Arthur R. Campman; Method of and Apparatus for Weaving Shaped Fabrics and Articles Woven Thereby; US Patent No. 313671; May 12, 1964.

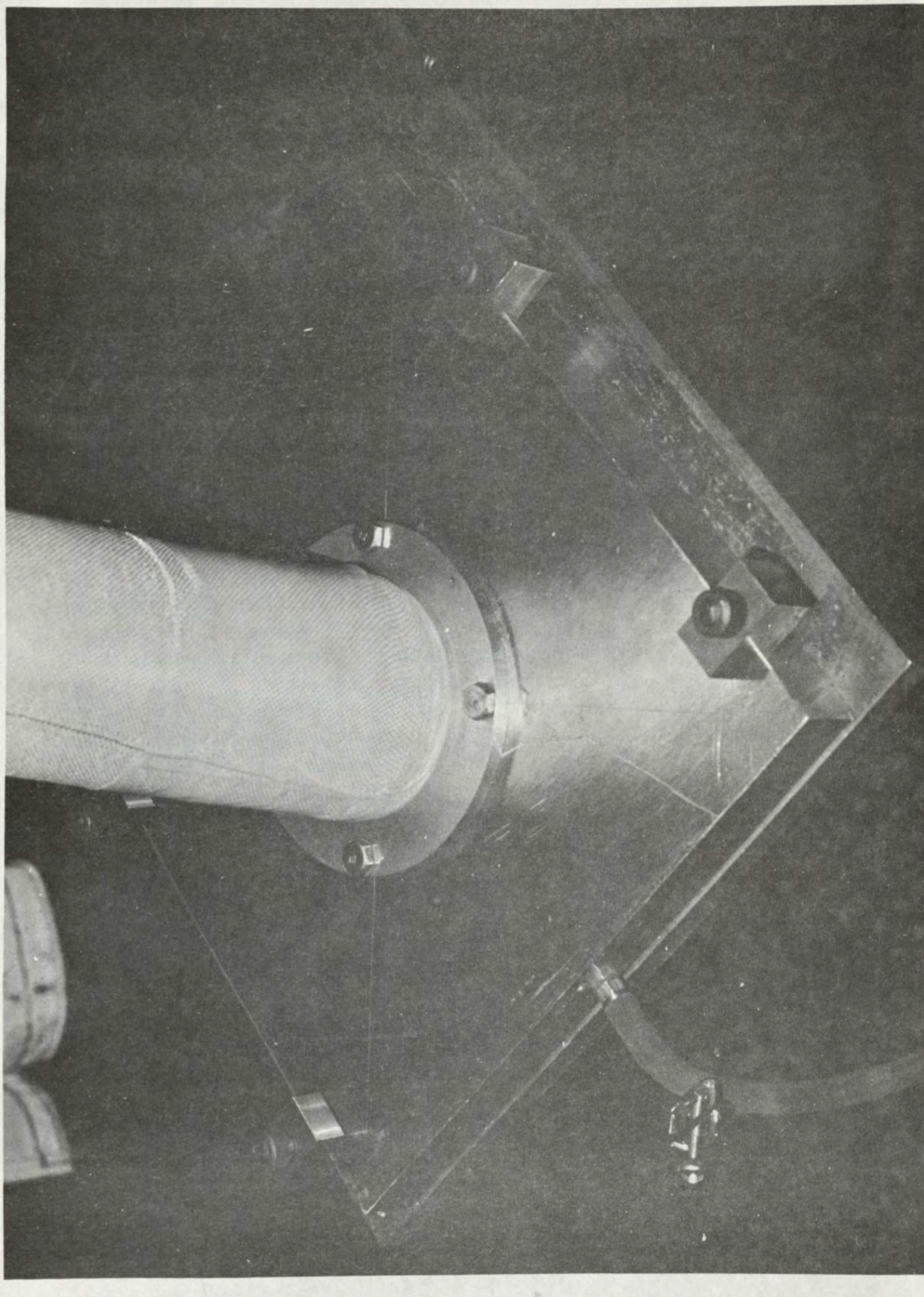


Figure 5. End Closure and Restraint for Fiberglass Arches.

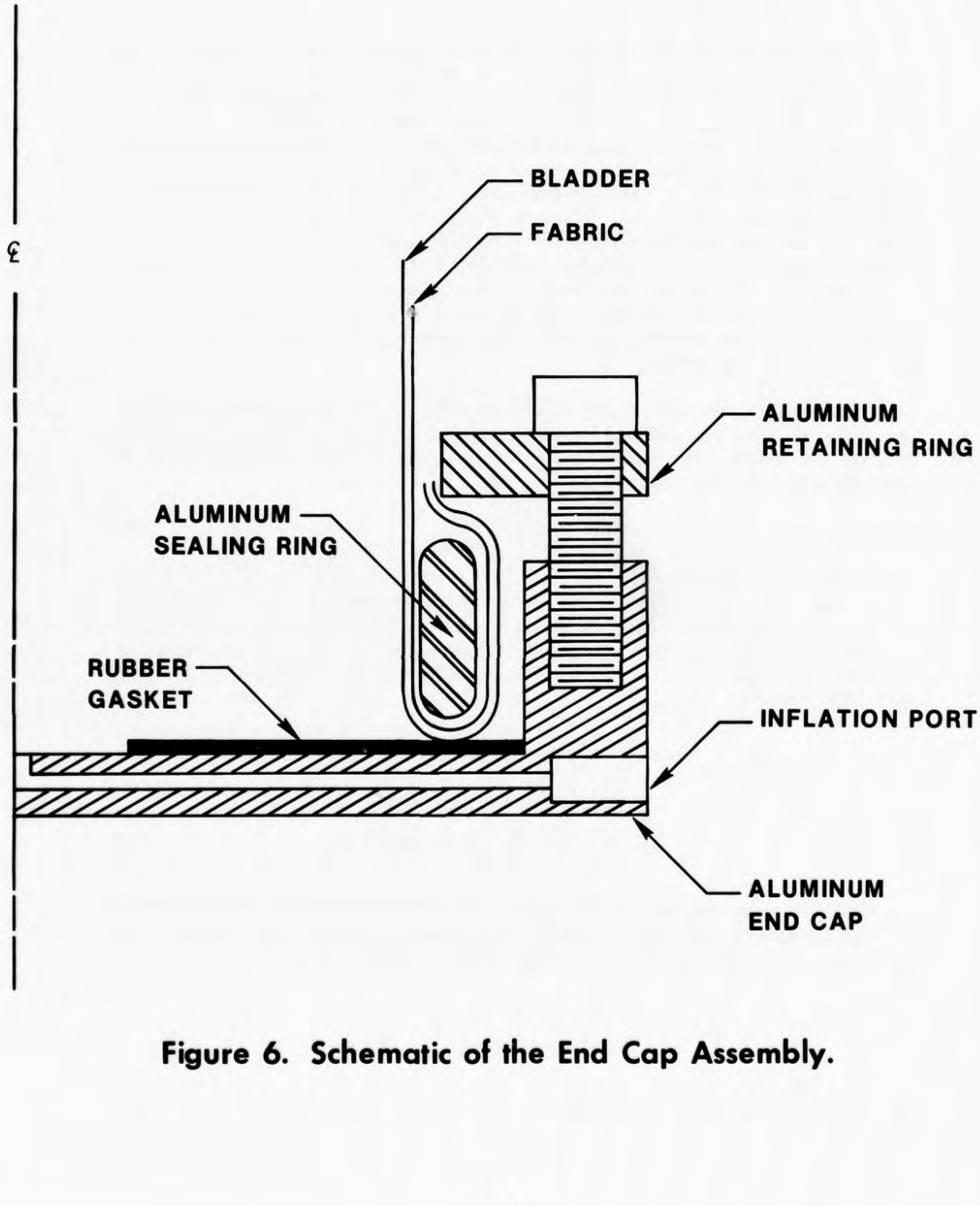


Figure 6. Schematic of the End Cap Assembly.

A schematic of the test apparatus is shown in Figure 7 and a photograph of the actual test apparatus is shown in Figure 8. The loading for all tests was a concentrated load at midspan. This load was applied using a 2.5-cm-wide strap wrapped around the arch. Tests were conducted with the load directed toward the arch center of curvature and away from the center of curvature. Wrinkling was, as expected, observed only when the load was directed toward the center of curvature, as this loading condition is the one which puts the arch in a compressive state of stress. The magnitude of the applied load was measured with a strain gage force transducer. The other measurement made was of the arch deflection at the point of the applied load. This was accomplished with a linear variable differential transformer positioned to measure the normal component of the deflection. The output from these transducers was amplified when required and recorded on a strip chart recorder, a sample of which is shown in Figure 9, for both the flexibility and the wrinkling load tests. Three tests were carried out for each pressure level with good repeatability.

To facilitate comparison of these experimental results with the theoretical prediction, the measured force and displacement were converted to the nondimensional parameters used in the theory. The measured normal component of displacement, W, is put in nondimensional form by the relation

$$\mathcal{W} = W/a \tag{47}$$

where $\mathcal{W}$ is the nondimensional displacement and a is the cross-section radius. Conversion of the measured force F_a to the nondimensional force parameters $\overline{g}_i$, i = 1,6 is illustrated in Figure 10. As shown in this figure, G_3 is the only nonvanishing component of the line load applied to the arch and this is in the negative direction. Thus the nondimensional force parameters of concern are $\overline{g}_1 = g_1/\overline{C}_{22}$ and $\overline{g}_2 = g_2/\overline{C}_{22}$. Using equations (25a) and (25c) we have

$$\overline{g}_1 = (1/\pi\overline{C}_{22})\int_0^{2\pi} G_3(\theta_1)\cos(\theta_1)d\theta_1 \tag{48}$$

$$\overline{g}_3 = -(1/\pi\overline{C}_{22})\int_0^{2\pi} G_3(\theta_1)\sin(\theta_1)d\theta_1 \tag{49}$$

where $\overline{C}_{22}$ is a reference value of the elastic modulus and the negative direction of the load $G_3(\theta_1)$ has been accounted for. Since $G_3(\theta_1)$ is nonvanishing only over a portion of the periphery of the arch, the limits and integrals can be changed to give

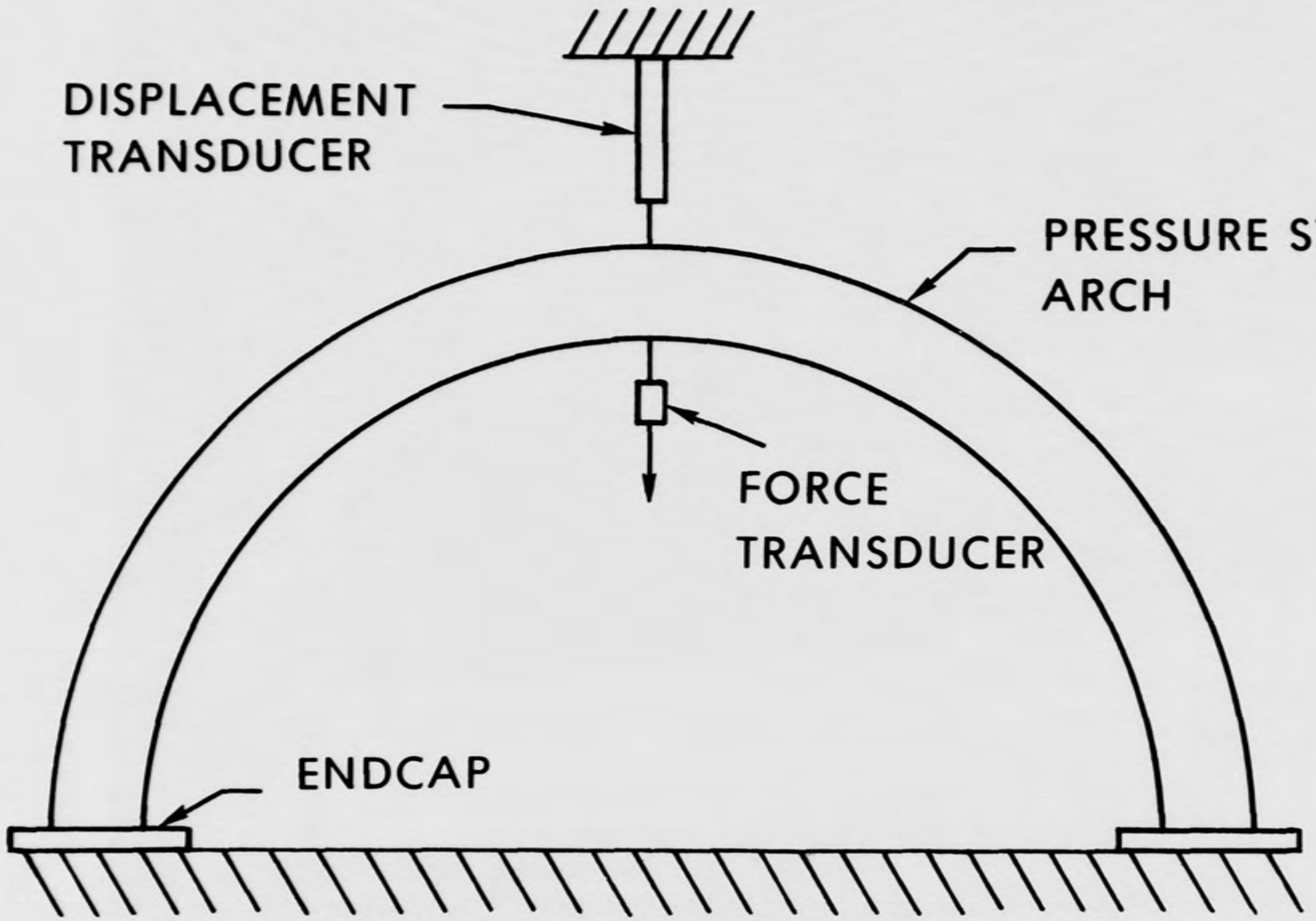


FIGURE 7. SCHEMATIC OF TEST APPARATUS

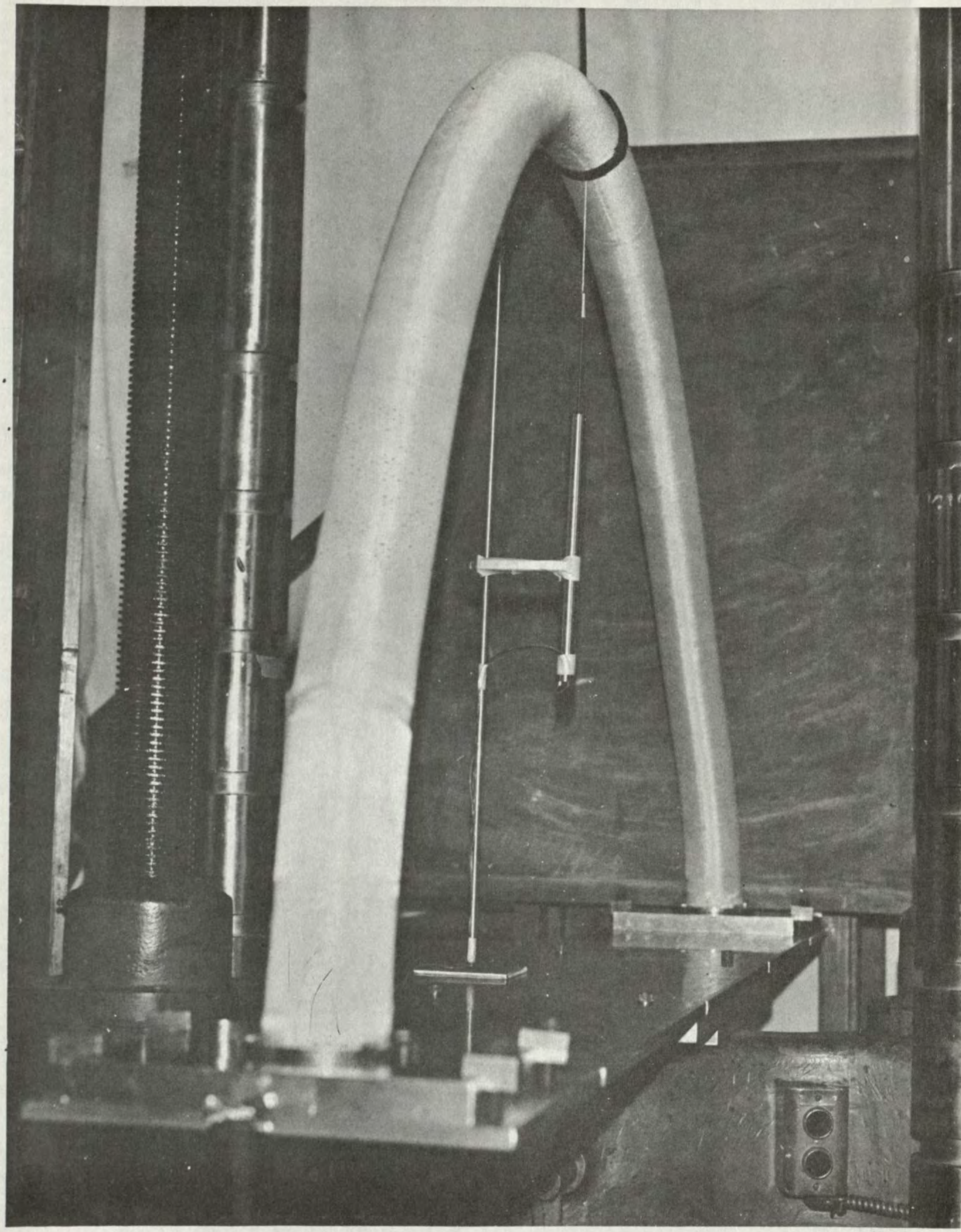


Figure 8. Arch Model in Test Apparatus.

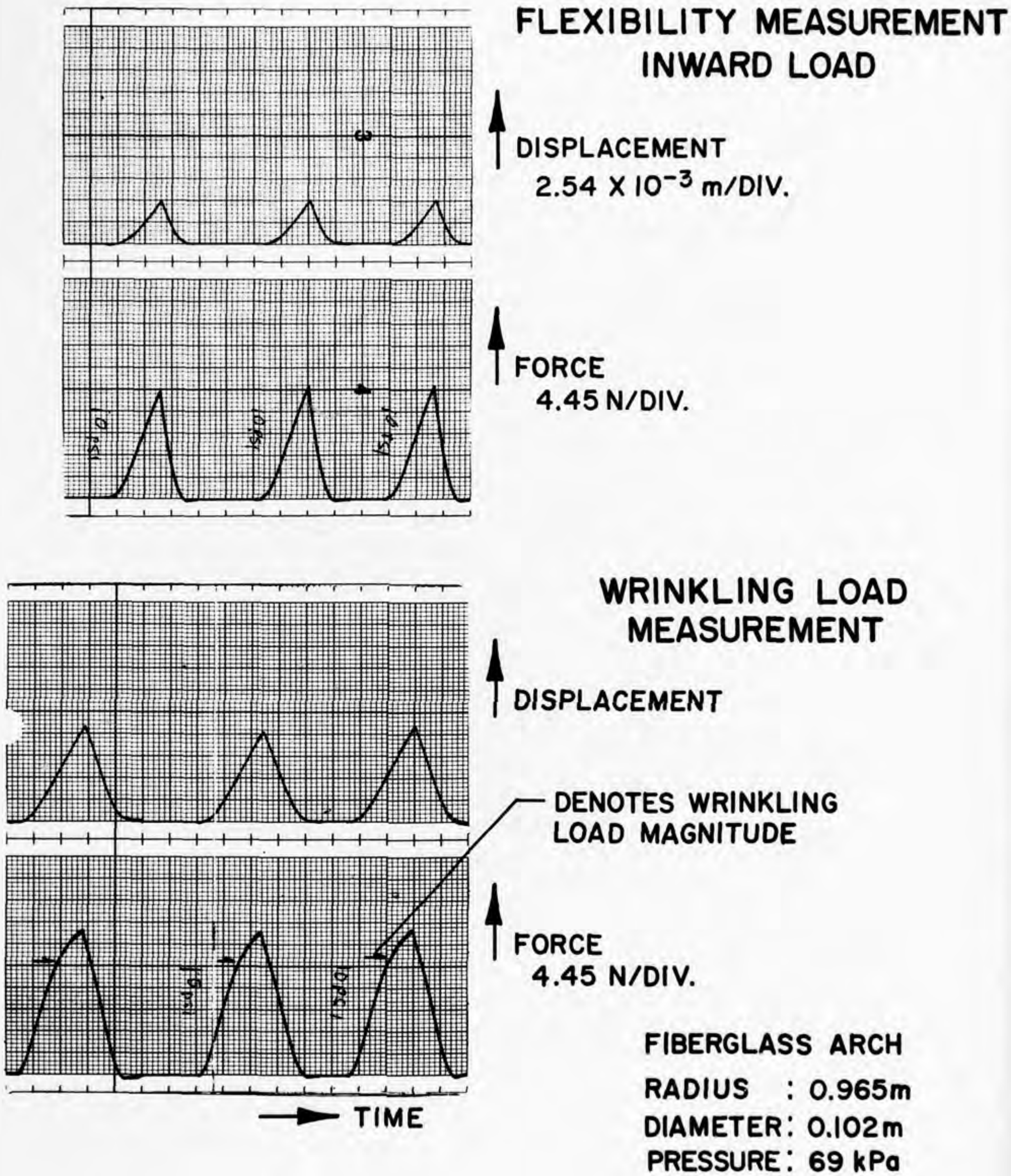


Figure 9. Typical Recordings of the Force and Displacement From Arch Loading Tests.

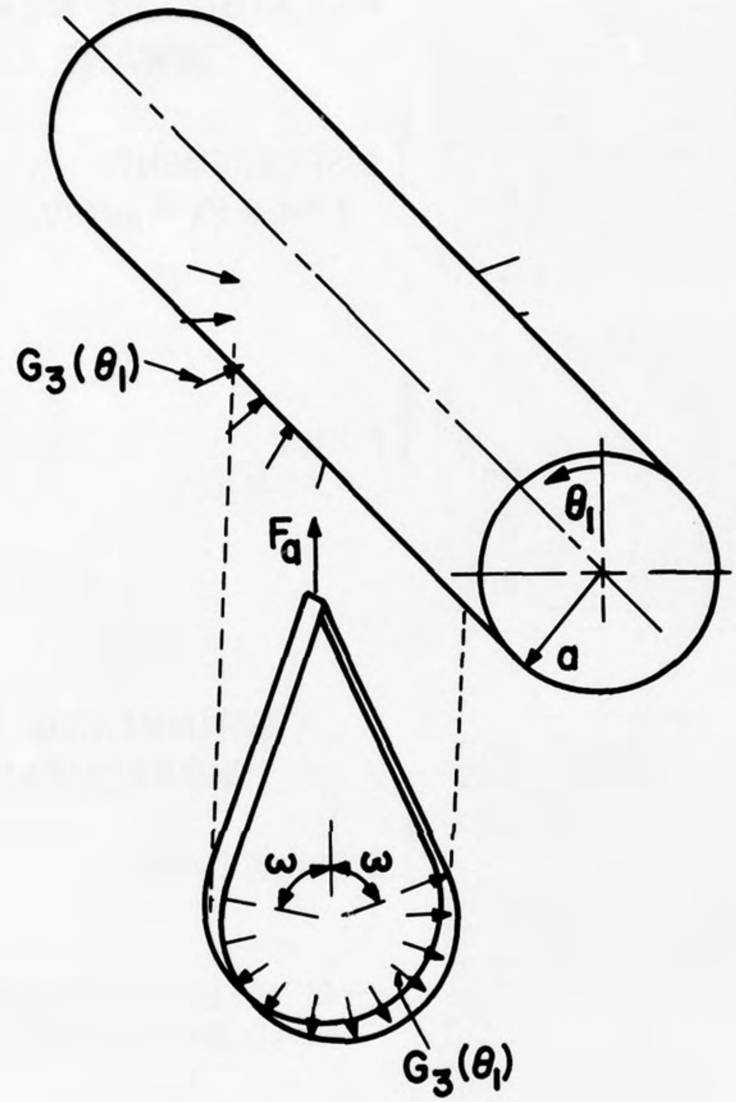


Figure 10. Arch Loading Technique

$$\bar{g}_1 = (1/\pi\bar{C}_{22})\int_{\omega}^{2\pi-\omega} G_3(\theta_1)\cos(\theta_1)d\theta_1 \tag{50}$$

$$\bar{g}_3 = -(1/\pi\bar{C}_{22})\int_{\omega}^{2\pi-\omega} G_3(\theta_1)\sin(\theta_1)d\theta_1 \tag{51}$$

Because of the symmetry of $G_3(\theta_1)$ about $\theta_1 = 0$, $\bar{g}_3$ vanishes and we need only be concerned with $\bar{g}_1$. To obtain a relationship between $\bar{g}_1$ and F_a we observe that $G_3(\theta_1)$ also acts on the webbing used to apply the load and for equilibrium must have

$$F_a = \int_{\omega}^{2\pi-\omega} G_3(\theta_1)\cos(\theta_1)a\,d\theta_1 \tag{52}$$

Substitution of (52) into (50) gives

$$\bar{g}_1 = F_a/\pi a\bar{C}_{22}$$

the expression used to convert the measured applied load to the nondimensional force parameter. If the subscript w denotes wrinkling then the nondimensional wrinkling load is given in terms of the physical wrinkling load as

$$\bar{g}_{1w} = F_{aw}/\pi a\bar{C}_{22} \tag{53}$$

the magnitude of F_{aw} is determined experimentally by finding the value of F_a for which the loading curve becomes nonlinear. As can be seen from Figure 9, the transition from linear behavior to nonlinear behavior is smooth, making the evaluation of F_{aw} rather difficult. This difficulty is especially pronounced in comparison to the behavior of pressure-stabilized beams which, as reported in reference 4, exhibit a constant load jump in deflection as wrinkling begins. With such behavior it was quite easy to evaluate the wrinkling load from the loading curve.

The flexibility is the transverse displacement due to a unit load which in nondimensional form is:

$$\gamma = W/g_1 \tag{54}$$

Substituting the expressions for W and $\bar{g}_1$ gives the following expression for the nondimensional flexibility in terms of the measured force and displacement:

$$\gamma = \pi\bar{C}_{22}W/F_a \tag{55}$$

Determination of Material Stiffness Properties

In order to accomplish the objective of comparing the theory with experimental results obtained by the above described procedure, the elastic and shear moduli of the fabric used in the arches must be measured. These moduli are needed for the theoretical computation of the behavior of these fiberglass arches. These moduli or stiffnesses were determined from the results of tension and torsion tests. Although these tests are very similar to those commonly used, they were performed on specimens in their pressurized state of stress. Testing in this manner yields stiffnesses relative to a state of stress very close to that present in the arch loading tests, that is, uniaxial tension or shear superimposed on biaxial tension resulting from internal pressure. The moduli were measured using relatively small excursions from the pressurized state of stress, and the behavior was assumed to be linear over these excursions. Thus, for a given value of pressure, the elastic and shear moduli are constants. However, because of the nonlinear behavior of fabrics, these moduli were found to be dependent on the pressure level. This testing procedure had the additional advantage of yielding the moduli of the fabric skin-polyethylene film composite, and although the bladder is thought to make little contribution to the stiffness, any contribution is included. It should also be recalled that the arches were woven with a natural curved contour, and it was not possible to devise a simple test using curved specimens that would be suitable for measuring the elastic and shear moduli. We therefore obtained some straight woven fabric tubes having a cross-section radius of 0.038 m for use in determination of the moduli in tension and torsion tests. These straight tubes were woven with the same fiberglass yarns and plain weave design used in the contoured arches so the stiffness characteristic of the straight tubes should closely represent that of the arches.

Elastic Modulus

As indicated above, the elastic modulus was determined from the results of a tension test, as is shown schematically in Figure 11. After pressurizing the specimen the test is performed in the usual manner, applying an axial force F and measuring its magnitude along with the corresponding elongation, e. The theory requires the constant C_{22} relating the stress resultant, N_{22}, and the strain ϵ_{22} as

$$N_{22} = C_{22}\epsilon_{22}$$

Thus, C_{22} is the slope of the stress resultant-strain curve and can be computed from the force-elongation curve generated from the tension test by the expression

$$C_{22} = l_g \Delta F / 2\pi a \Delta e$$

where ΔF and Δe are respectively increments of applied force and specimen elongation. In the tests conducted, the specimen gage length, l_g, and the radius, a, were 0.343 m and 0.038 m, respectively. Typical force-elongation plots from these tests are shown in Figure 12.

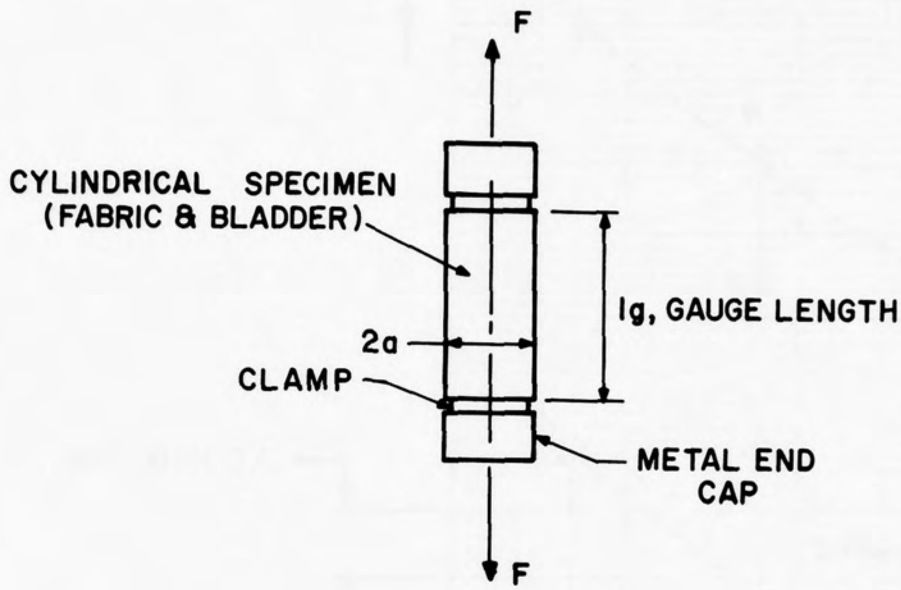
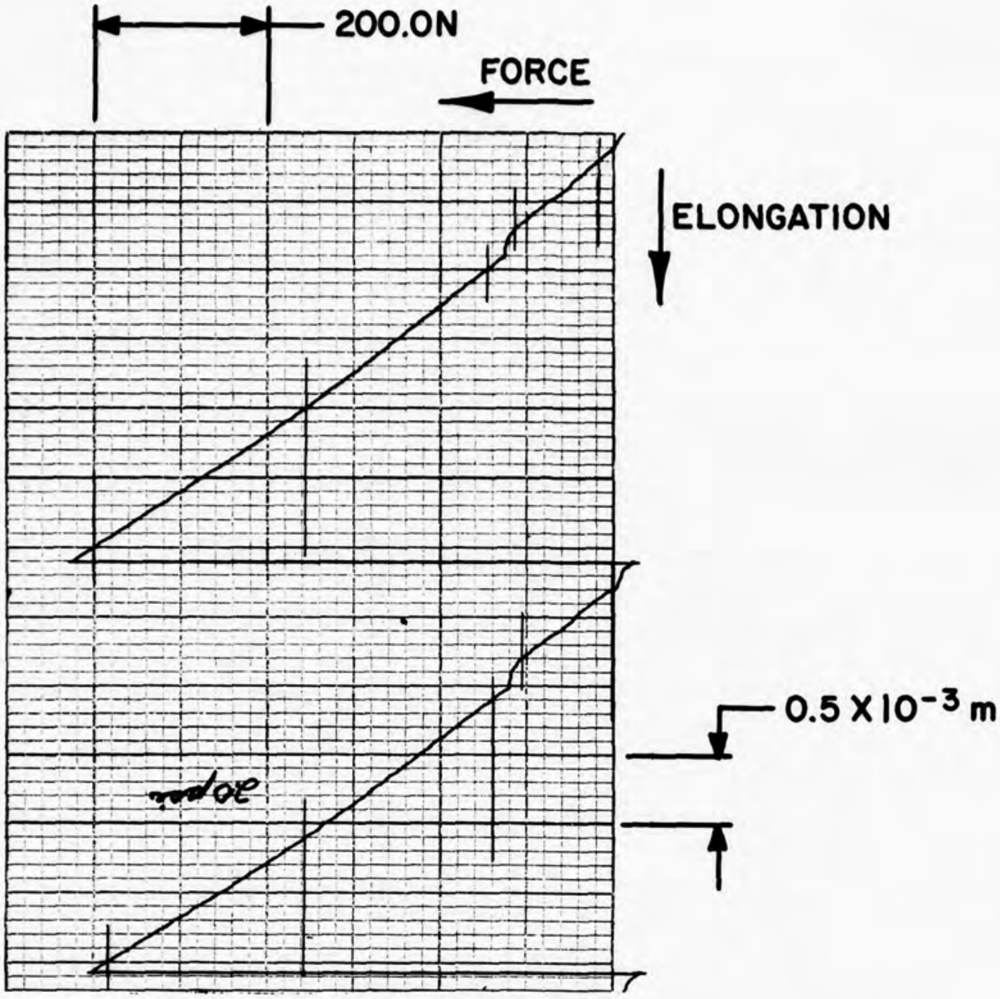


Figure 11. Schematic Diagram of the Tension Test



FIBERGLASS FABRIC WITH 4 MIL POLYETHYLENE BLADDER

GAUGE LENGTH = 0.343m
RADIUS = 0.038m
PRESSURE = 138,000 Pa

Figure 12. Typical Force-Elongation Recordings From Tension Test.

Shear Modulus

The shear modulus was determined from the results of a torsion test, as shown schematically in Figure 13. This test is performed by inflating the specimen to a given pressure and applying a torque, T, about the axis of the cylinder and measuring ϕ, the rotation of the end of the cylinder. The theory requires the constant, C_{33}, relating the shear stress resultant, N_{12}, to the shear strain, ϵ_{12}, as

$$N_{12} = C_{33}\epsilon_{12}$$

The parameter C_{33} is the slope of the stress resultant-strain curve and can be computed from the torque-rotation curve generated from a torsion test by the expression

$$C_{33} = l_g \Delta T / 2\pi a^3 \Delta\phi$$

where ΔT and $\Delta\phi$ are, respectively, increments of applied torque and specimen rotation. The parameters, l_g and, a are as defined above and for the tests conducted have the values 0.267 m and 0.038 m, respectively. Typical torque and rotation plots from these tests are shown in Figure 14.

RESULTS AND DISCUSSION

In this section we will present the experimental results obtained with the procedures just described and compare the arch performance characteristics obtained experimentally with those predicted by the theory developed in this report.

Material Properties

The material properties required by the theory are the elastic modulus and the shear modulus. As is indicated above, these parameters are found as a function of pressure, as shown in Figures 15 and 16. In these figures are plotted the nondimensional elastic and shear moduli as a function of the nondimensional pressure parameter. These are the nondimensional parameters used in the theory with the value of the elastic modulus, C_{22}, for zero pressure being used as the reference value. It is denoted as $\bar{C}_{22}$. The pressure parameter which is used throughout this report is the nondimensional axial stress resultant due to internal pressure and is given in terms of the pressure, P, and the cross-section radius, a, as:

$$n = Pa/2\bar{C}_{22}$$

where n denotes the pressure parameter. In both Figures 15 and 16 the experimental results which are tabulated in Tables 1 and 2 are denoted by the symbols, and the least squares linear fits of the data used for the theoretical prediction of the arch behavior are shown both graphically and mathematically. The reference value of the elastic modulus, $\bar{C}_{22}$, is also given.

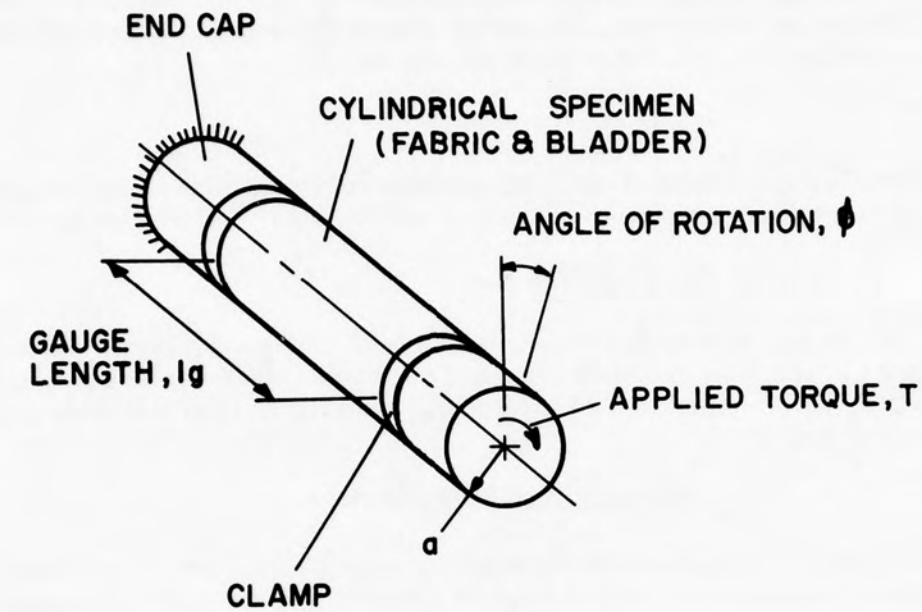


Figure 13. Schematic Diagram of the Torsion Test

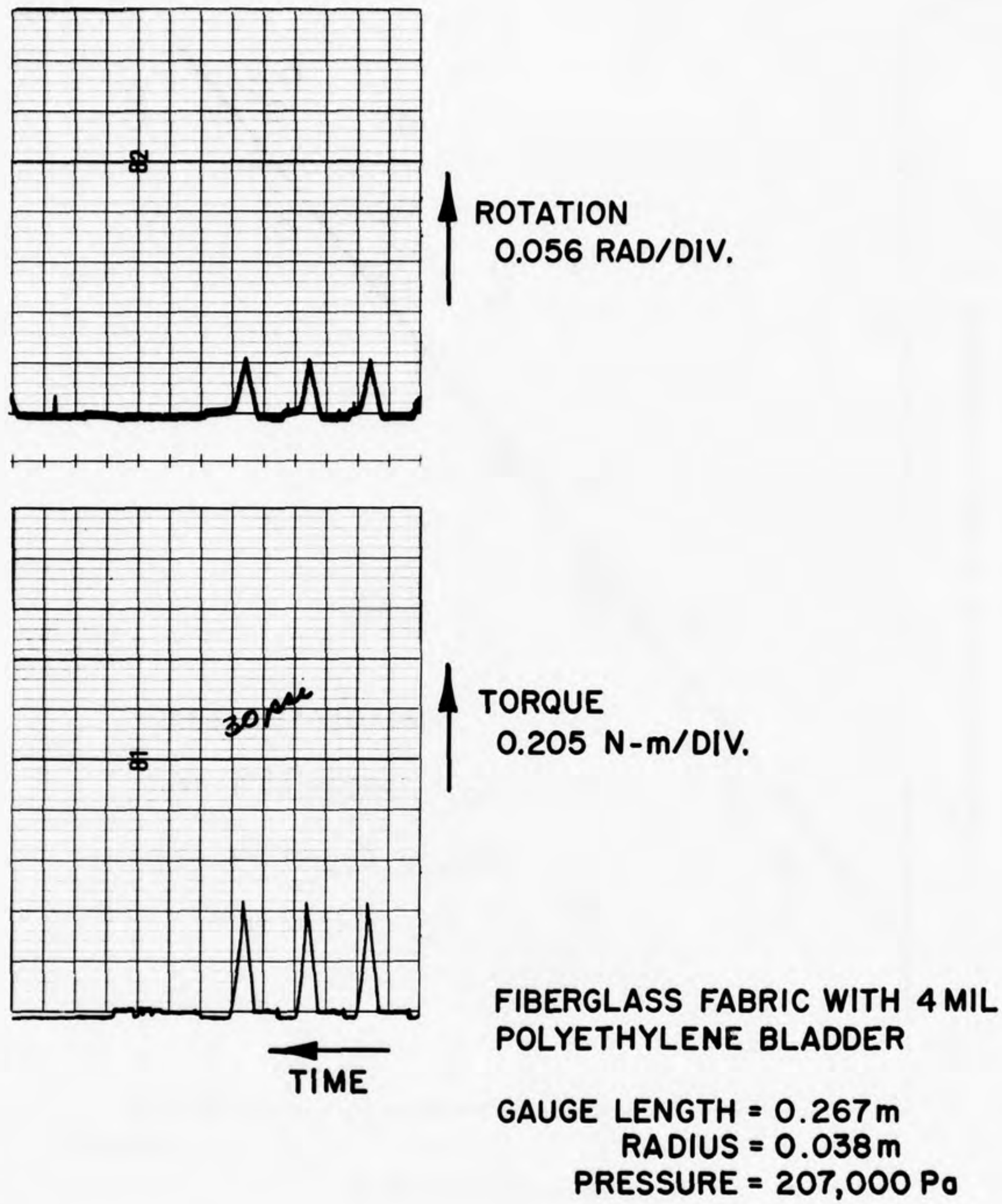


Figure 14. Typical Torque and Rotation Recordings From Torsion Test.

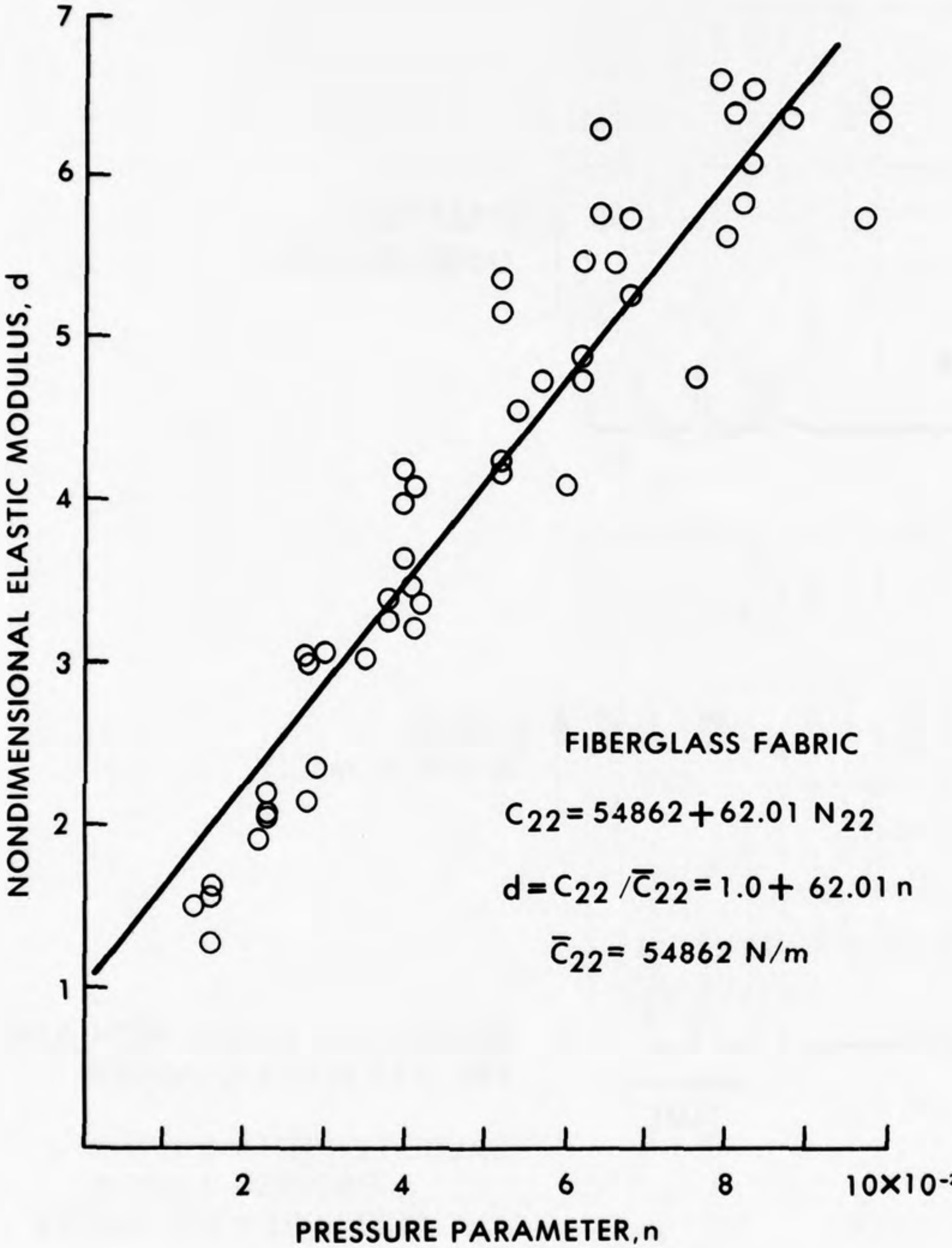


FIGURE 15. ELASTIC MODULUS AS A FUNCTION OF THE PRESSURE PARAMETER

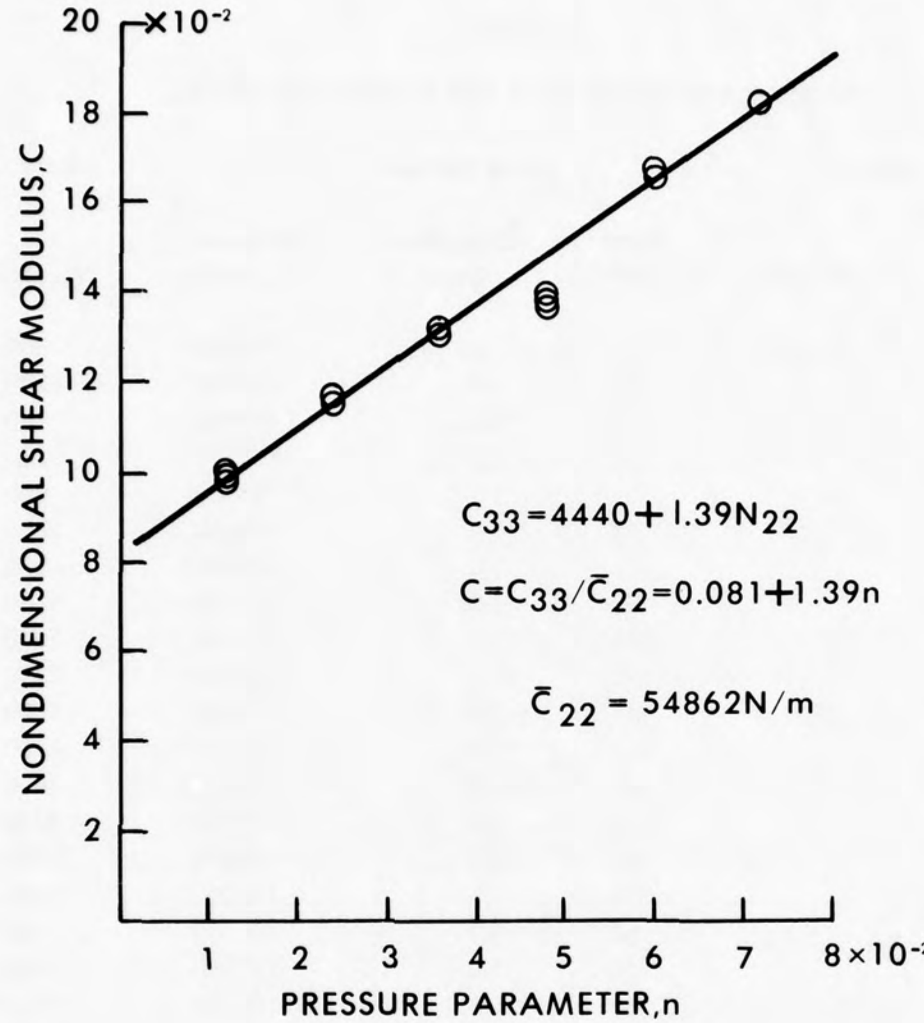


FIGURE 16. SHEAR MODULUS AS A FUNCTION OF THE PRESSURE PARAMETER

TABLE 1

ELASTIC STIFFNESS DATA FOR FIBERGLASS FABRIC

Pressure		Elastic Stiffness			Stress
P, kPa	n, Nondim	Force ΔF, N	Elongation Δe, m	Modulus C_{22} = N/m	N_{22}, N/m
34.5	0.012	49.	1.0×10^{-3}	70400	864
		58.	0.8	104100	1200
		57.	1.0	81900	780
		63.	0.8	113100	1280
		59.	1.0	84800	905
		79.	1.0	113500	1280
		61.	1.0	87600	864
		83.	1.0	119200	1280
		115.	1.0	165200	1910
		162.	1.0	232700	2830
69.0	0.024	81.	1.0	116400	1520
		122.	1.0	175300	2270
		94.	0.9	150000	1600
		132.	1.0	189600	2230
		82.	0.7	168300	1560
		138.	1.0	198200	2190
		80.	0.7	164100	1520
		124.	1.0	178100	2100
		81.	0.7	166200	1520
		129.	1.0	185300	2100
		81.	0.7	166200	1520
		128.	1.0	183900	2100
		173.	1.0	248500	2980
		182.	1.0	261400	4200

TABLE 1

ELASTIC STIFFNESS DATA FOR FIBERGLASS FABRIC (cont'd)

Pressure		Elastic Stiffness			Stress
P, kPa	n, Nondim	Force ΔF, N	Elongation Δe, m	Modulus C_{22}, N/m	N_{22}, N/m
103.5	0.036	64.	0.5×10^{-3}	183900	2300
		156.	1.0	224100	3300
		76.	0.5	218400	2220
		180.	1.0	258600	3390
		80.	0.5	229800	2220
		186.	1.0	267200	3390
		78.	0.5	224100	2260
		180.	1.0	258600	3140
		214.	1.0	307400	4390
138.0	0.048	80.	0.5	229800	2870
		200.	1.0	287300	3710
		200.	0.9	319200	4500
		98.	0.5	281600	2870
		208.	1.0	298800	3620
		243.	1.0	349100	4630
		102.	0.5	293100	2870
		218.	1.0	313200	3710
		224.	0.9	357600	4550
172.5	0.060	120.	0.5	344800	3530
		232.	1.0	333300	4530
		218.	1.0	313200	5370
		104.	0.5	298800	3380
		244.	1.0	350500	4450
		242.	1.0	347600	5450
		88.	0.4	316000	3530
		252.	1.0	362000	4360
		248.	1.0	356300	5450

TABLE 2

SHEAR STIFFNESS DATA FOR FIBERGLASS FABRIC

Pressure		Shear Stiffness		
P, kPa	n, Nondim.	Torque ΔT, N–m	Rotation $\Delta\phi$, rad.	Modulus C_{33}, N/m
34.5	0.012	2.32	0.319	5584.
		2.17	0.305	5510.
		2.32	0.346	5192.
69.0	0.024	2.82	0.346	6312.
		3.15	0.388	6287.
		2.71	0.332	6312.
103.5	0.036	3.11	0.332	7254.
		3.08	0.332	7184.
		3.11	0.332	7254.
138.0	0.048	3.13	0.319	7599.
		3.22	0.332	7511.
		3.58	0.360	7701.
172.5	0.060	3.94	0.332	9190.
		3.76	0.319	9128.
		3.66	0.305	9293.
207.0	0.072	3.93	0.305	9979.
		3.93	0.305	9979.
		3.93	0.305	9979.

In reducing the data for the elastic modulus, increments of axial force and elongation were taken from two or three segments of the plot as indicated in Figure 12. Once the modulus was computed using these increments a decision had to be made regarding the value of the pressure parameter or, equivalently, axial stress associated with this modulus. In previous work, reference 4, the pressure parameter, or axial stress due to pressure at which the test was run, was used. In those tests, however, only a short segment of the load-elongation curve beyond the pressurized state was used. In the present work, as indicated above, several segments beyond the pressurized state were used, and it was decided to use a corrected pressure parameter equal to the sum of the axial stress due to pressure and the axial stress due to axial load at the beginning of the increment used to calculate the modulus. This corrected value of the pressure parameter is given in the last column of Table 1 which is headed "Stress." The data from Table 1 is plotted on Figure 15 using this corrected pressure parameter.

The data in Figure 15 appears to have some curvature indicating that a higher order curve might provide a better fit. However, it was decided that in the present context a linear approximation would be sufficiently accurate.

It is interesting to compare the values of elastic stiffness used in the analysis with the limiting value obtained by considering the yarns as axial springs in parallel. The axial stiffness of a fiberglass yarn is given as 28.2 N/Tex. Thus for 512 yarns of 133 Tex in parallel, a stiffness of 1.92×10^6 N/m is obtained. The maximum value of stiffness used in the analysis is 0.62×10^6 N/m, for the 0.089-m cross-section radius at 207 kPa pressure. Thus the stiffnesses were all less than one-third of the limiting stiffness. The fiberglass was chosen for this work because it has a linear stress-strain behavior, and it was believed that this would simplify the analysis problems. As can be seen from the data shown here, the resulting fabric is far from linear. The stress-strain behavior exhibits a stiffening effect as stress is increased, which is typical of fabrics. Over the stress range shown in Figure 15 the stiffness increases by a factor of nearly seven. This is attributed to the yarn and fabric structure because fiberglass does have a linear stress-strain behavior in comparison with other materials used in fabrics such as nylon and dacron. This increase in stiffness of the fiberglass fabric is a marked contrast to very small increases for a dacron fabric and the absence of increases in a nylon fabric reported in reference 4. These facts illustrate some of the difficulties in testing fabrics as continuum structures.

The results from the torsion test are presented in Figure 16 in terms of a plot of the shear modulus as a function of the nondimensional pressure which is, as noted above, the nondimensional axial stress. These results are rather unremarkable, showing a shear stiffness of 10 to 18 percent of the reference elastic stiffness or modulus. These percentages are somewhat larger than those of the nylon and dacron fabrics of reference 4. Those fabrics had shear stiffness of 3 to 6 percent and 4 to 12 percent, respectively, of their reference elastic stiffness. However, the elastic stiffness of the nylon and dacron fabrics were found to be approximately three times the elastic stiffness of the present fiberglass fabric, so on an absolute basis the three fabrics have nearly equal shear stiffness.

Behavior of Arches Under Load

The results from the arch loading tests along with companion theoretical predictions are given in graphical form on Figures 17 through 24 for the four arches described earlier. The experimental data in numerical form on which the plots are based is given in Tables 3 through 6. For each arch a plot for both the flexibility and the wrinkling load as a function of pressure are included with the experimental results denoted by symbols and the theoretical predictions by the curves. The flexibility is defined as the deformation per unit applied force. The wrinkling load is the load for which the maximum compressive stress resulting from the applied load is equal in magnitude to the tensile stress resulting from pressurization. Since the fabric cannot resist compressive stress, any further increase in the applied load would cause wrinkling of the fabric. The flexibility and the wrinkling load are then, respectively, measures of the deformation behavior and the load-carrying capability.

Flexibility

The results for the arch flexibility are presented in Figures 17, 19, 21 and 23. Experimental results are given for the flexibility with the applied force directed inward, that is, toward the center of curvature of the arch and outward, away from the center of curvature. One would not expect these two cases to be significantly different, but for the arch with a radius ratio of 25.0, the flexibility for the inward force is significantly larger than that for the outward force. A similar situation exists for the arch having a radius ratio of 11.3 except the differences are not as great in this case. For the other two arches, the difference is of the order of magnitude of the experimental error. In general, the difference in the flexibility resulting from the change in loading direction is greatest at low pressures and becomes significantly smaller with increasing pressure. It will be observed that the flexibilities associated with the inward force are greater than those associated with the outward force except for the arch with a radius ratio of 13.0 and for this arch the opposite is true, but the differences are also the smallest for this case. A possible explanation of this behavior is a differenital stiffness effect caused by the stress or internal force due to the applied load. In the arch, in contrast to the straight beam, lateral deformation is accompanied by an internal axial force which is positive for the outward force and negative for the inward force. This internal force is analogous to the internal force resulting from the pressure and if it is large enough can have the same influence. The pressure internal force is positive and decreases the flexibility of the arch. Therefore, if the internal force due to applied load were to have this same effect, it would be expected that under the outward force the arch would be less flexible than under the inward force, and this is what is observed. This phenomenon is not included in the theory since only the internal axial force due to pressure has an effect on the stiffness or flexibility of the arch. It should also be pointed out that this phenomenon is of no aid in explaining the disagreement between theory and experiment.

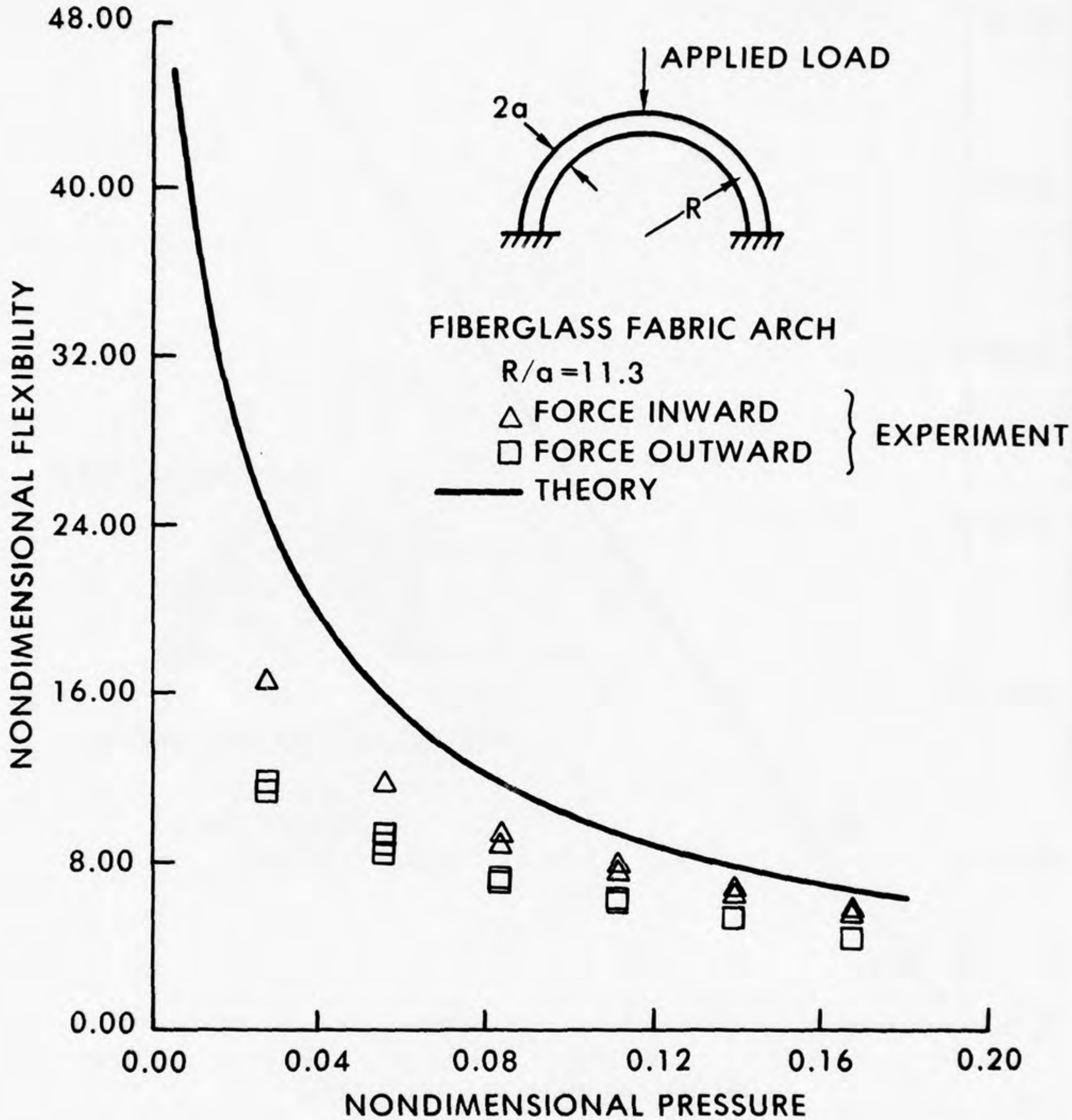


FIGURE 17. ARCH FLEXIBILITY AS A FUNCTION OF PRESSURE

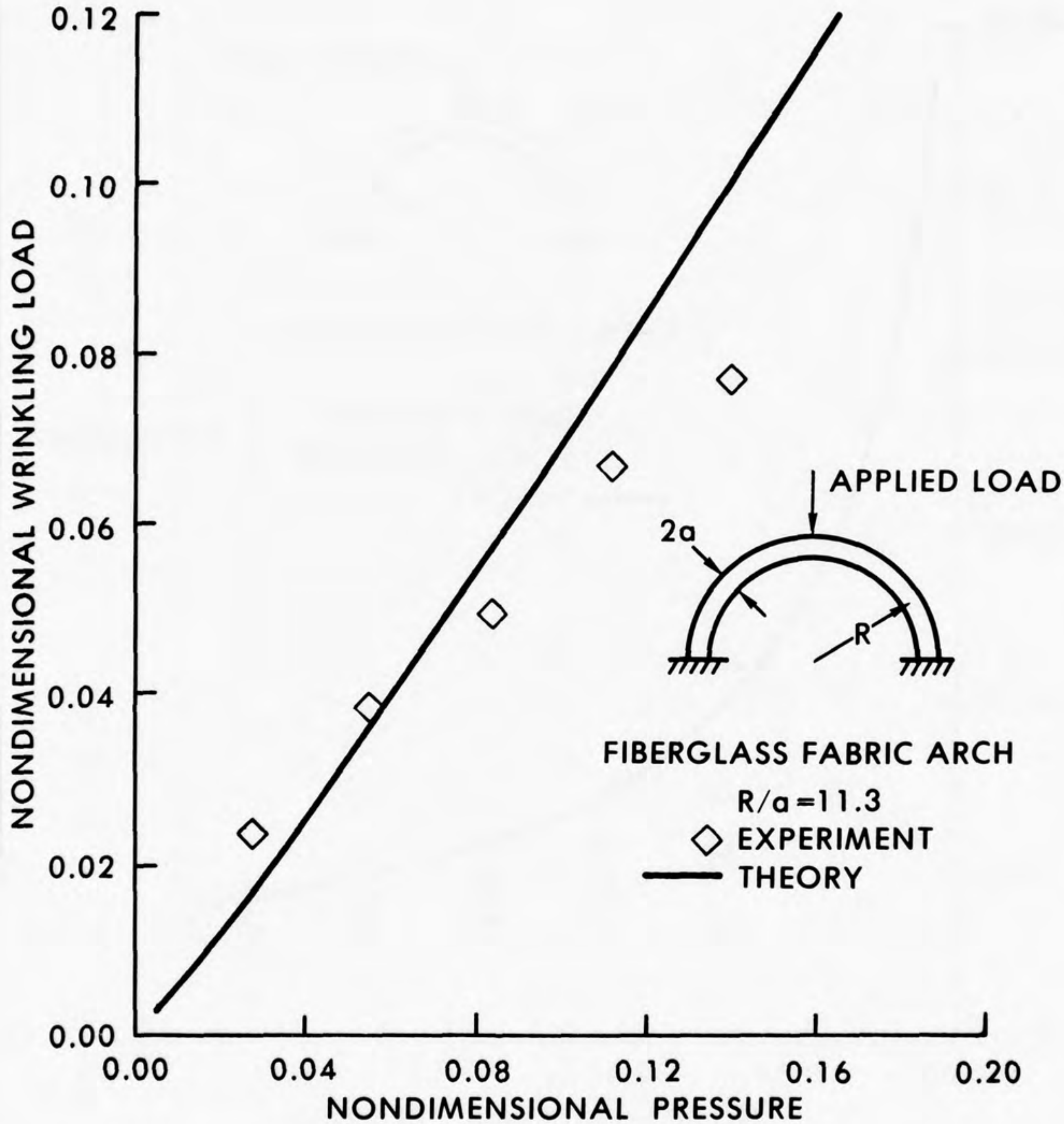


FIGURE 18. ARCH WRINKLING LOAD AS A FUNCTION OF PRESSURE

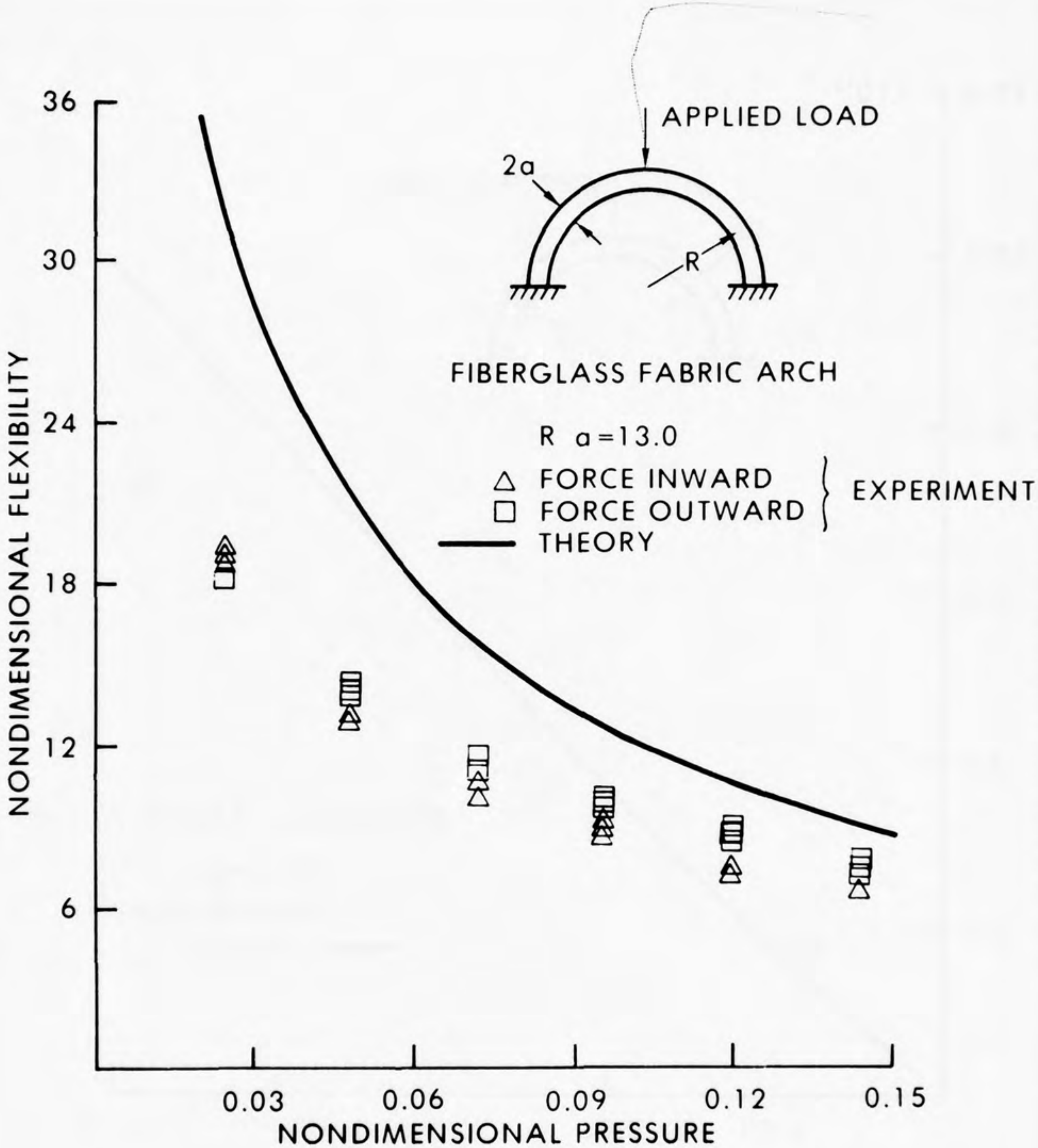


FIGURE 19. ARCH FLEXIBILITY AS A FUNCTION OF PRESSURE

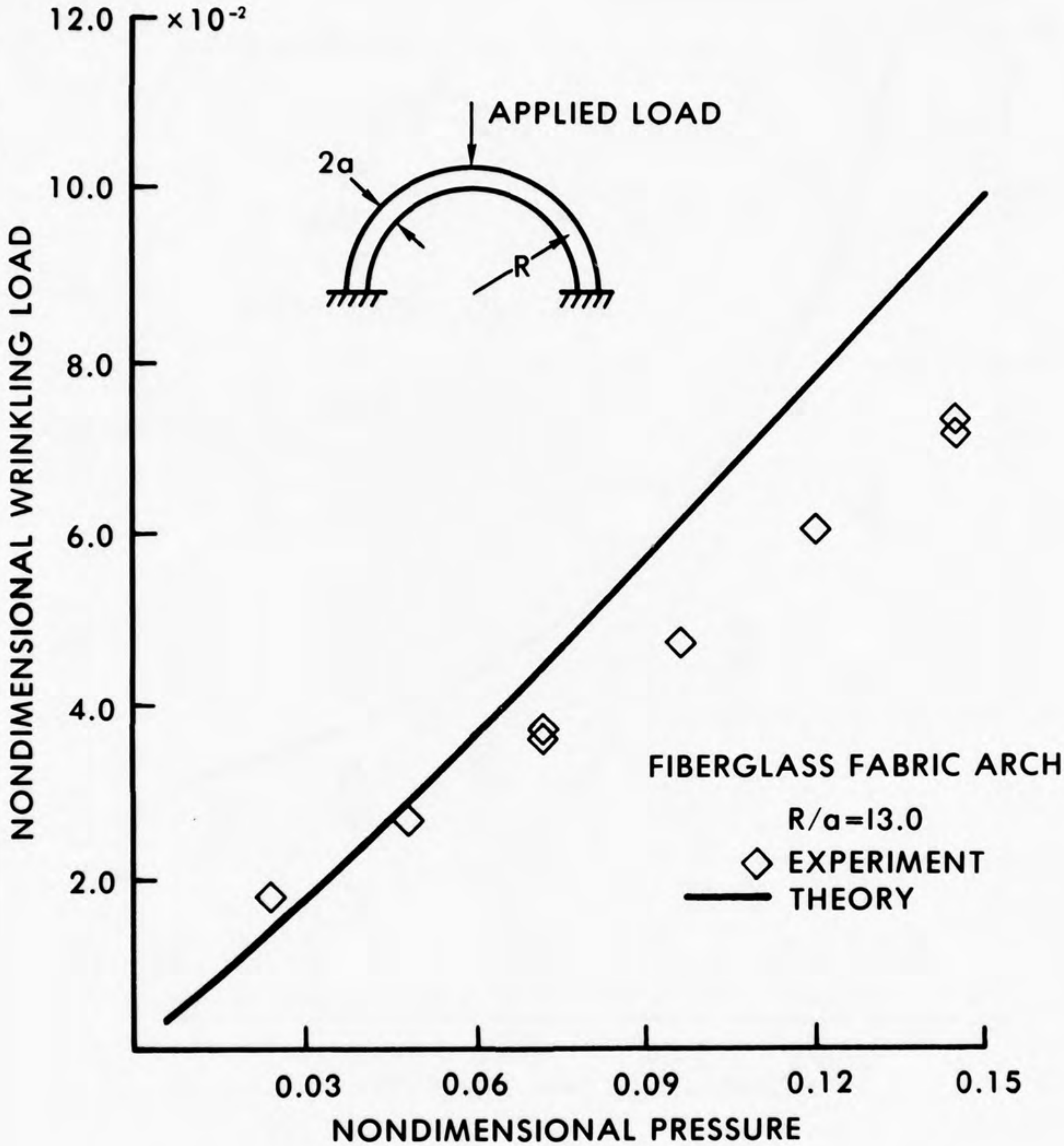


FIGURE 20. ARCH WRINKLING LOAD AS A FUNCTION OF PRESSURE

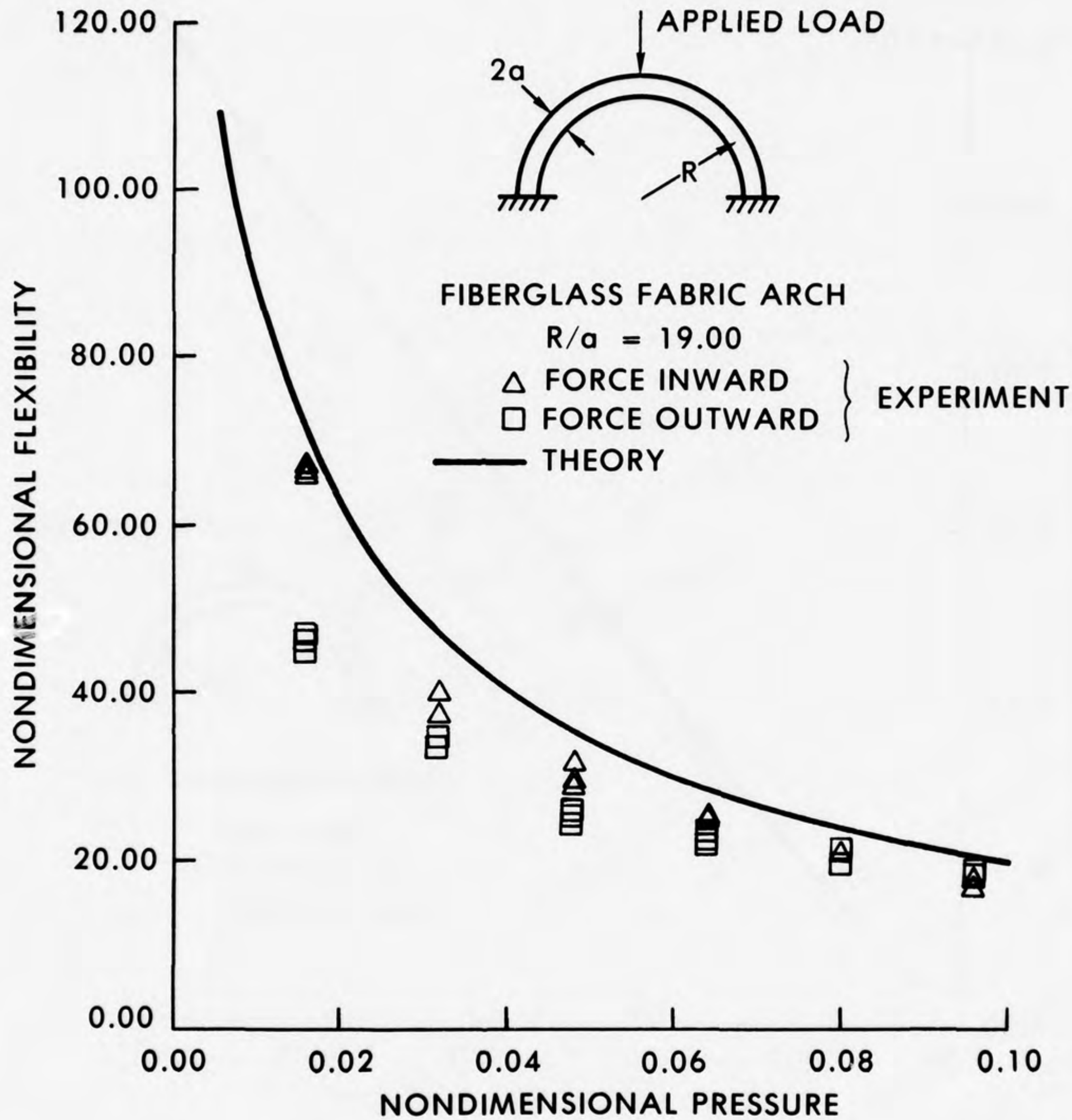


FIGURE 21. ARCH FLEXIBILITY AS A FUNCTION OF PRESSURE

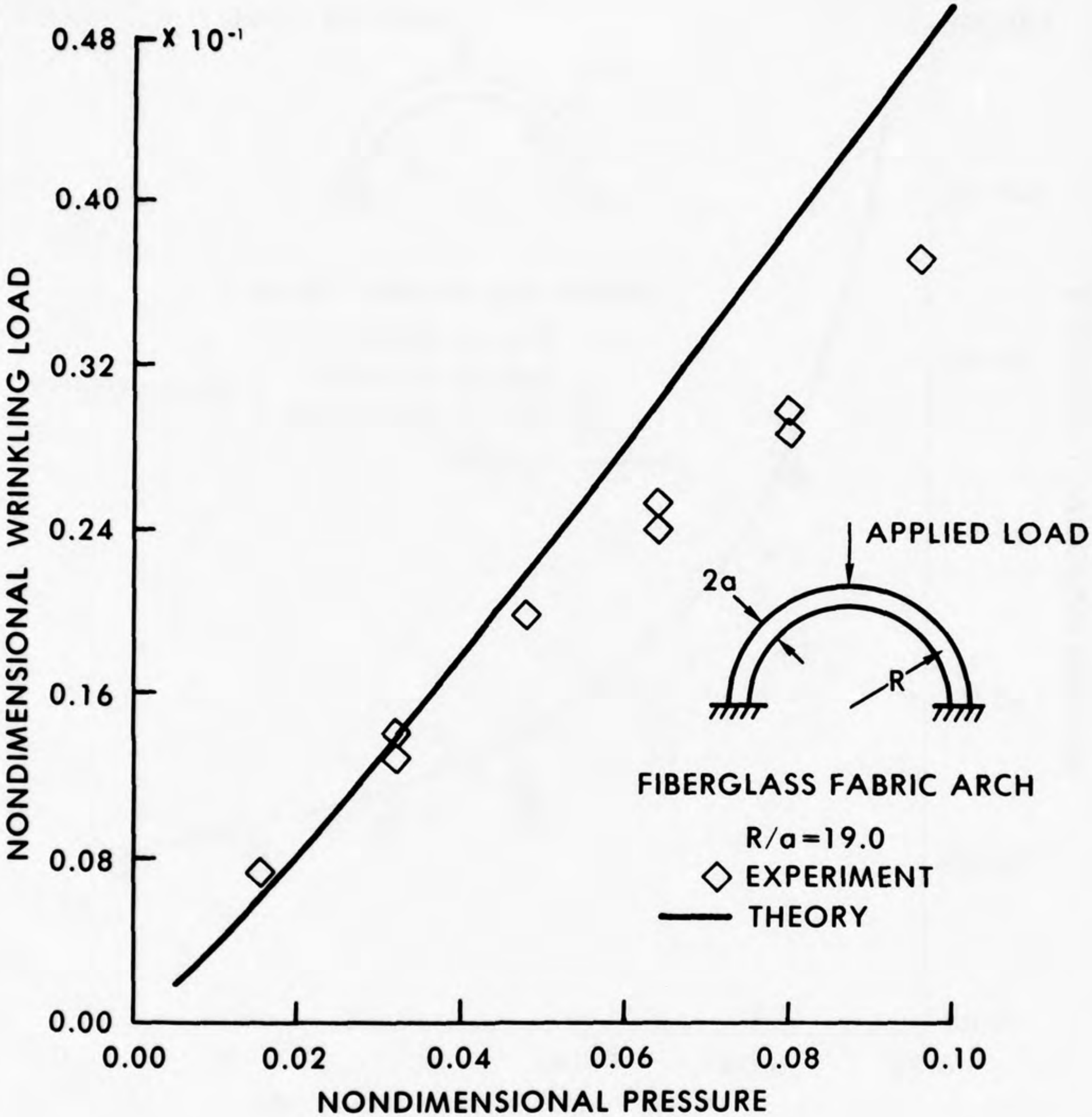


FIGURE 22. ARCH WRINKLING LOAD AS A FUNCTION OF PRESSURE

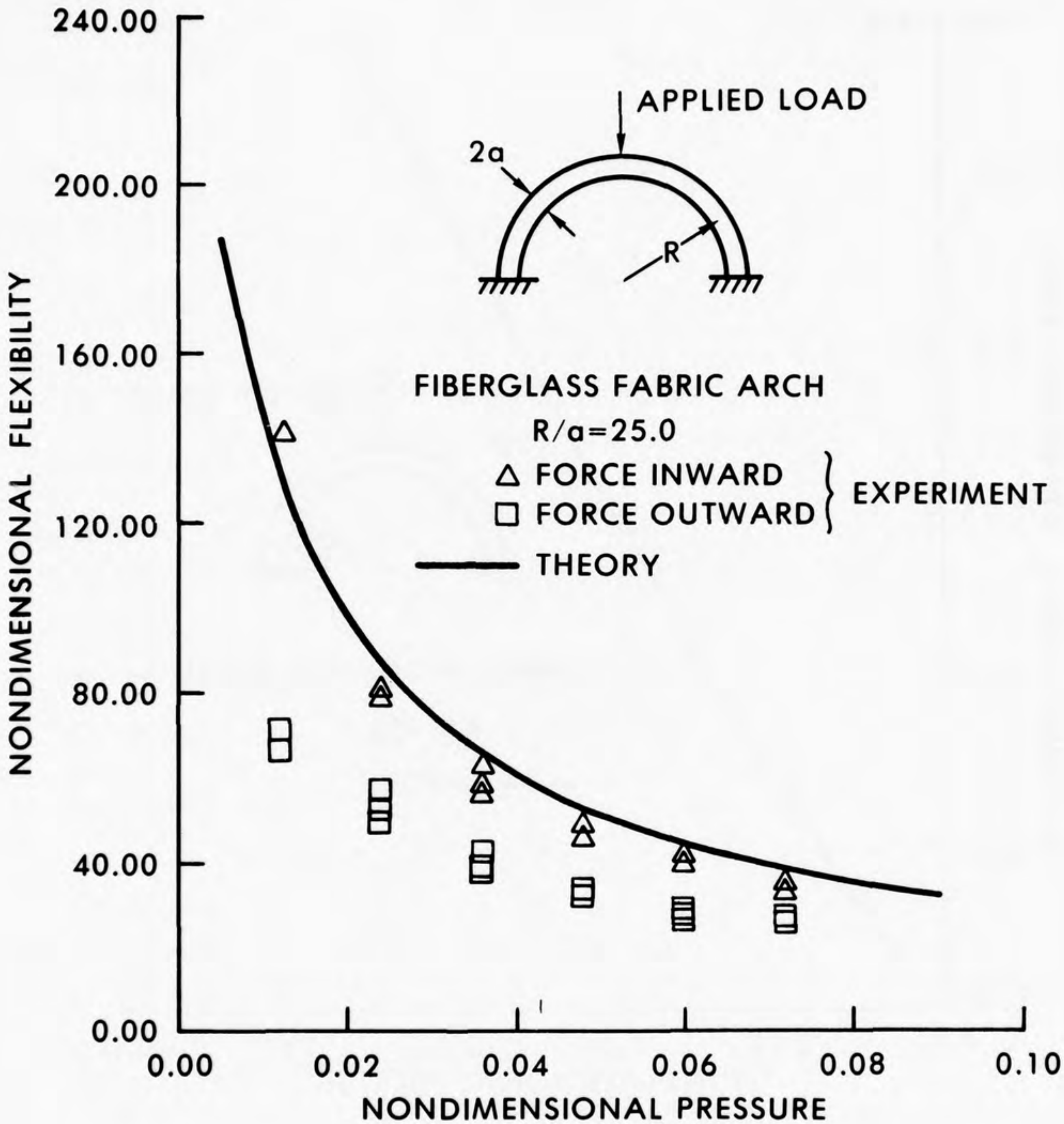


FIGURE 23. ARCH FLEXIBILITY AS A FUNCTION OF PRESSURE

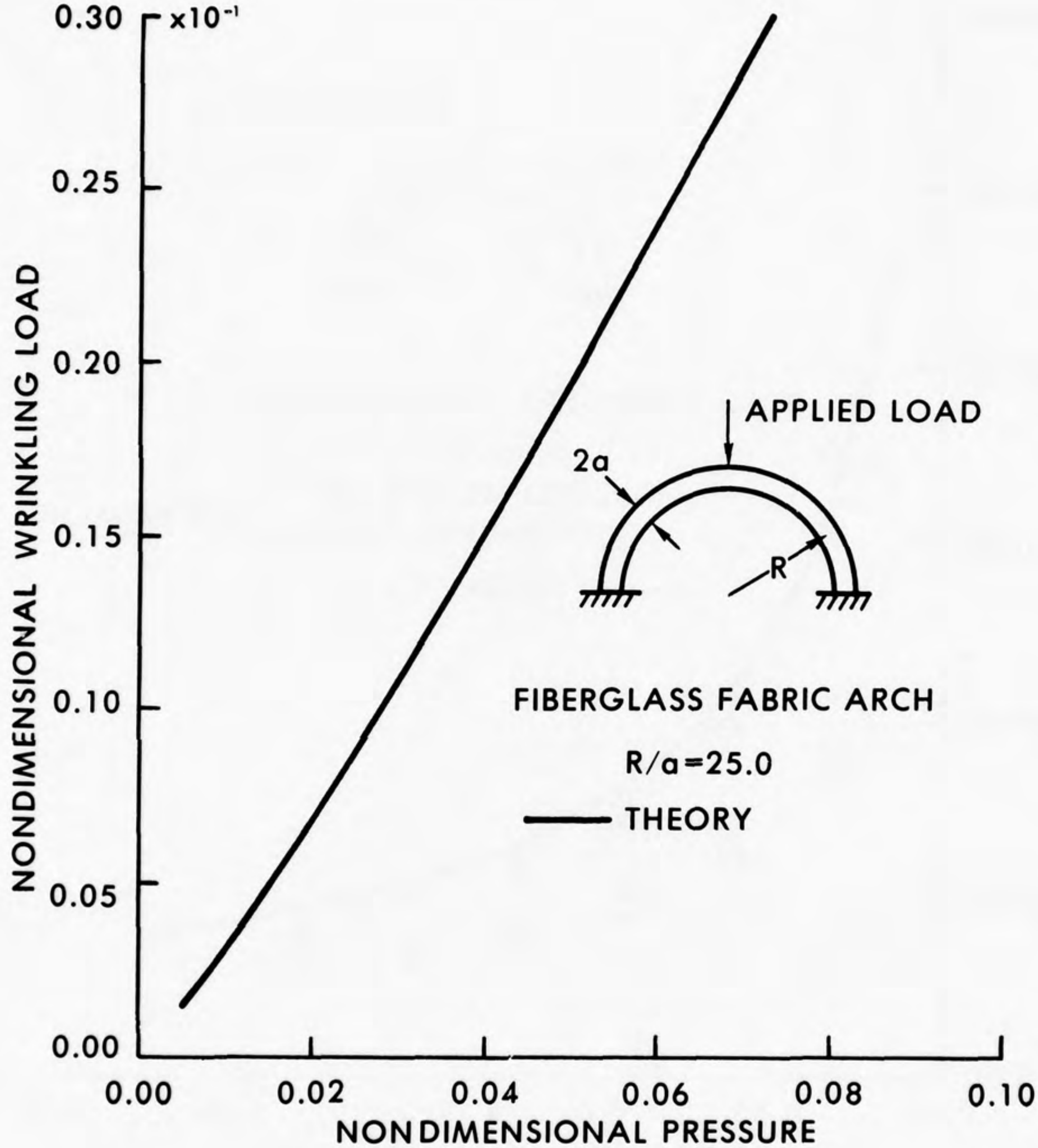


FIGURE 24. ARCH WRINKLING LOAD AS A FUNCTION OF PRESSURE

TABLE 3

TEST DATA FOR ARCH WITH ρ = 11.3

Flexibility

Pressure		Force Outward		Force Inward		Wrinkling Load	
P, kPa	n, Nondim.	w/Fa, m/N	γ, Nondim.	w/Fa, m/N	γ, Nondim.	Faw, N	gw, Nondim.
34.5	0.028	6.79×10^{-5}	11.7	9.59×10^{-5}	16.5	365.	2.38×10^{-2}
		6.79	11.7	9.59	16.5	365.	2.38
		6.62	11.4	9.59	16.5	365.	2.38
69.0	0.056	5.31	9.15	6.74	11.6	587.	3.83
		5.08	8.75	6.79	11.7	587.	3.83
		4.91	8.46	6.74	11.6	587.	3.83
103.5	0.084	4.28	7.38	5.31	9.15	756.	4.93
		4.22	7.27	5.08	8.76	756.	4.93
		4.11	7.08	5.02	8.65	756.	4.93
138.0	0.112	3.71	6.39	4.51	7.77	1023.	6.67
		3.65	6.29	4.34	7.48	1023.	6.67
		3.60	6.20	4.28	7.38	1023.	6.67
172.5	0.140	3.25	5.60	3.88	6.69	1179.	7.69
		3.20	5.52	3.71	6.39	1179.	7.69
		3.20	5.52	3.71	6.39	1179.	7.69
207.0	0.168	2.85	4.91	3.43	5.91		
		2.85	4.91	3.43	5.91		
		2.85	4.91	3.48	5.91		

TABLE 4

TEST DATA FOR ARCH WITH ρ = 13.0

Pressure		Flexibility				Wrinkling Load	
		Force Outward		Force Inward			
P, kPa	n, Nondim.	w/Fa, m/N	γ, Nondim.	w/Fa, m/N	γ, Nondim.	Faw, N	gw, Nondim.
34.5	0.024	10.9×10^{-5}	18.8	11.4×10^{-5}	19.6	231.	1.76×10^{-2}
		10.9	18.8	10.9	18.8	231.	1.76
		10.9	18.8	11.2	19.3		
69.0	0.048	8.62	14.8	7.94	13.7	356.	2.72
		8.39	14.5	7.82	13.5	356.	2.72
		8.11	14.0	7.82	13.5		
103.5	0.072	6.62	11.4	6.22	10.7	476.	3.63
		6.91	11.9	6.05	10.4	467.	3.56
		6.91	11.9	6.05	10.4		
138.0	0.096	5.94	10.2	5.31	9.15	600.	4.58
		5.82	10.0	5.08	8.75	600.	4.58
		5.71	9.84	5.19	8.94		
172.5	0.120	5.25	9.05	4.39	7.57	756.	5.77
		5.08	8.75	4.28	7.38	756.	5.77
		4.91	8.46	4.28	7.38		
207.0	0.144	4.57	7.88	3.88	6.69	934.	7.13
		4.45	7.67	3.83	6.60	912.	6.96
		4.44	7.65	3.83	6.60	912.	6.96

TABLE 5

TEST DATA FOR ARCH WITH $\rho = 19$

Pressure		Flexibility				Wrinkling Load	
		Force Outward		Force Inward			
P, kPa	n, Nondim.	w/Fa, m/N	γ, Nondim.	w/Fa, m/N	γ, m/N	Faw, N	gw, Nondim.
34.5	0.016	26.9×10^{-5}	46.4	38.6×10^{-5}	66.5	62.3	0.71×10^{-2}
		27.2	46.9	38.8	66.9	62.3	0.71
		26.2	45.1	38.3	66.0	62.3	0.71
69.0	0.032	20.7	35.7	22.9	39.5	116.	1.32
		20.1	34.6	22.0	37.9	116.	1.32
		20.5	35.3	22.0	37.9	120.	1.36
103.5	0.048	16.1	27.7	18.1	31.2	173.	1.97
		15.9	27.4	17.5	30.2	173.	1.97
		15.7	27.1	17.3	29.8	173.	1.97
138.0	0.064	13.8	23.8	14.8	25.5	218.	2.48
		13.6	23.4	14.6	25.2	213.	2.42
		13.3	22.9	14.4	24.8	213.	2.42
172.5	0.080	12.2	21.0	12.4	21.4	253.	2.88
		11.9	20.5	12.2	21.0	258.	2.93
		11.8	20.3	12.1	20.8	253.	2.88
207.0	0.096	11.1	19.1	10.4	17.9	325.	3.70
		10.8	18.6	10.2	17.6	325.	3.70
		10.7	18.4	9.99	17.2	325.	3.70

TABLE 6

TEST DATA FOR ARCH WITH ρ = 25

Pressure		Flexibility				Wrinkling Load	
		Force Outward		Force Inward			
P, kPa	n, Nondim.	w/Fa, m/N	γ, Nondim.	w/Fa, m/N	γ, Nondim.	Faw, N	gw, Nondim.
34.5	0.012	40.8 $\times 10^{-5}$	70.3	81.6 $\times 10^{-5}$	140.6		
		39.1	67.4	81.6	140.6		
		39.1	67.4	81.6	140.6		
69.0	0.024	32.8	56.5	45.7	78.8		
		30.7	52.9	46.4	80.0		
		29.4	50.7	46.4	80.0		
103.5	0.036	24.2	41.7	37.3	64.3		
		22.8	39.3	35.5	61.2		
		22.4	38.6	35.9	61.9		
138.0	0.048	19.3	33.3	29.0	50.0		
		18.7	32.2	27.9	48.1		
		18.7	32.2	27.7	47.7		
172.5	0.060	16.3	28.1	24.8	42.7		
		15.6	26.9	23.6	40.7		
		15.9	27.4	23.8	41.0		
207.0	0.072	15.6	26.9	21.0	36.2		
		14.8	25.5	20.4	35.2		
		14.8	25.5	20.3	35.0		

Both the theory and experiment show the same general behavior of the flexibility with pressure. Much like that found for pressure-stabilized beams, the flexibility of the arch varies inversely with the pressure. The theory is in better agreement with the experimental results for the inward force, and the quality of the agreement ranges from *good* over the entire pressure range for the arches with radius ratios of 25.0 and 19.0 to *poor* for the other two arches at the lower pressures. In general, agreement between theory and experiment is better at higher pressures and for larger radius ratios. The theory predicts the arches to be more flexible than they are found to be experimentally. This is somewhat unexpected because we are dealing with fixed end restraints, and it is always an experimental problem to attain full fixity of the ends. Since fixity in the theory can by attained perfectly, it is expected that the theoretical prediction will be less flexible than the experimental results. This anomaly, along with other differences between theory and experiment, may result from the treatment of the nonlinear stress-strain behavior of the fiberglass fabric, that is, linearizing it about the pressurized state. An additional factor that could contribute to the lack of agreement between theory and experiment is the use of straight woven tubes to measure the fabric moduli or stiffnesses used in the theoretical representation of the contoured arch behavior.

Wrinkling Load

The wrinkling load results are presented in Figures 18, 20, 22 and 24 which show the wrinkling as determined both experimentally and theoretically, increasing monotonically with pressure. In the lower pressure range the increase in the wrinkling load is seen to be more rapid than a linear function, but at higher pressures the behavior is essentially linear. It should be noted that these experimental results are for the applied force directed inward, since this loading puts the arch in a compressive state of stress, thus making wrinkling possible. Experimental wrinkling load data were not collected for the arch having a radius ratio of 25.0 because of experimental difficulties in getting the arch inflated without distortion. The fiberglass fabric was very easily distorted when not inflated and any such distortion had to be corrected as the arch was inflated. This difficulty was more pronounced with the arches having the larger radius ratios. For the arch with radius ratio equal to 25.0 we were able to inflate the arch satisfactorily for the flexibility tests but could not inflate it without distortion for the wrinkling load determination. The theory is in good agreement with the experimental results in the low and middle pressure ranges but beyond this middle range the agreement becomes poorer as the pressure increases. It is speculated that, as with the flexibility, the disagreement is partly due to the linearization of the stress-strain behavior of the fabric and the use of straight fiberglass fabric tube to determine the stiffnesses used to represent the three-dimensionally woven fabric arches. As will be shown below, increases in stiffness decrease the wrinkling load; therefore, the disagreement between theory and experiment for the wrinkling load is consistent with that for the flexibility. This work was begun under the assumption that the mode of failure would be wrinkling and not geometric instability, a possible failure mode for arch structures. The conduct of the experiments confirmed this assumption. The wrinkle formation could not be observed because it was covered by the strap used in the load application. However, the failure was a gradual one with no

out-of-plane deformation. This is in contrast to the character of instability failures which occur rapidly and are accompanied by out-of-plane deformation. In addition, the agreement between the theoretical prediction and the experimental determination of the wrinkling load substantiates the belief that the experimental failures occurred by the wrinkling mode and not as the result of instability.

It will be recalled that the elastic modulus of the fiberglass fabric used in these arches varied widely with the pressure parameter. This variation has an impact on the load-carrying capability of the arches as measured by the wrinkling load. The nondimensional wrinkling load is computed from the theory by the relation

$$g_w = n/\epsilon_m E(n)$$

where g_w is the nondimensional wrinkling load, n is the pressure parameter, ϵ_m is the absolute value of the maximum compressive strain and E(n) is the nondimensional elastic modulus, a function of the pressure parameter. If E(n) is a constant, that is, not a function of pressure, the wrinkling load increases with pressure by two factors; first, the increase in pressure itself and secondly the decrease in ϵ_m as a result of the increased stiffness caused by the pressure increase. When the elastic modulus is a function of the pressure there are two competing effects. The strain ϵ_m tends to decrease because of the stiffness increase due to both increases in pressure and the corresponding increase in E(n). This decrease in strain magnitude is, however, diminished by the increased magnitude of E(n). This situation is shown graphically in Figure 25, where we have a plot of the wrinkling load as a function of pressure. The different curves in this plot represent the behavior of the wrinkling load for different rates of increase of the elastic modulus. The modulus is a linear function of the pressure parameter, and the slope of this linear function is the parameter that changes in Figure 25. Thus the curve $\sigma = 0$ represents the case where E(n) is a constant and that for $\sigma = 62.0$ is the behavior of the arch described in this report. The remaining curve is for intermediate rate of increase. It is clear from these results that the increase in the stiffness more than overcomes the decrease in strain resulting in a net loss in load-carrying capability. Thus to obtain the most benefit in load-carrying capability from increases in pressure, it is desirable to have the smallest increase in elastic stiffness due to pressure of the fabric used to make the arch.

These results illustrate the deformation and load-carrying capability of pressure-stabilized arches, and the comparisons of the experimental and theoretical results give confidence that the theory can predict the bahavior. The theory can then serve as a useful design tool.

CONCLUDING REMARKS

The development of a linear theory for the behavior of pressure-stabilized arches under static load has been presented and solutions to the resulting governing equations obtained. Solutions were obtained in the form of a Green's Function from which solutions to general loading can be obtained. In addition, a particular solution for the uniform

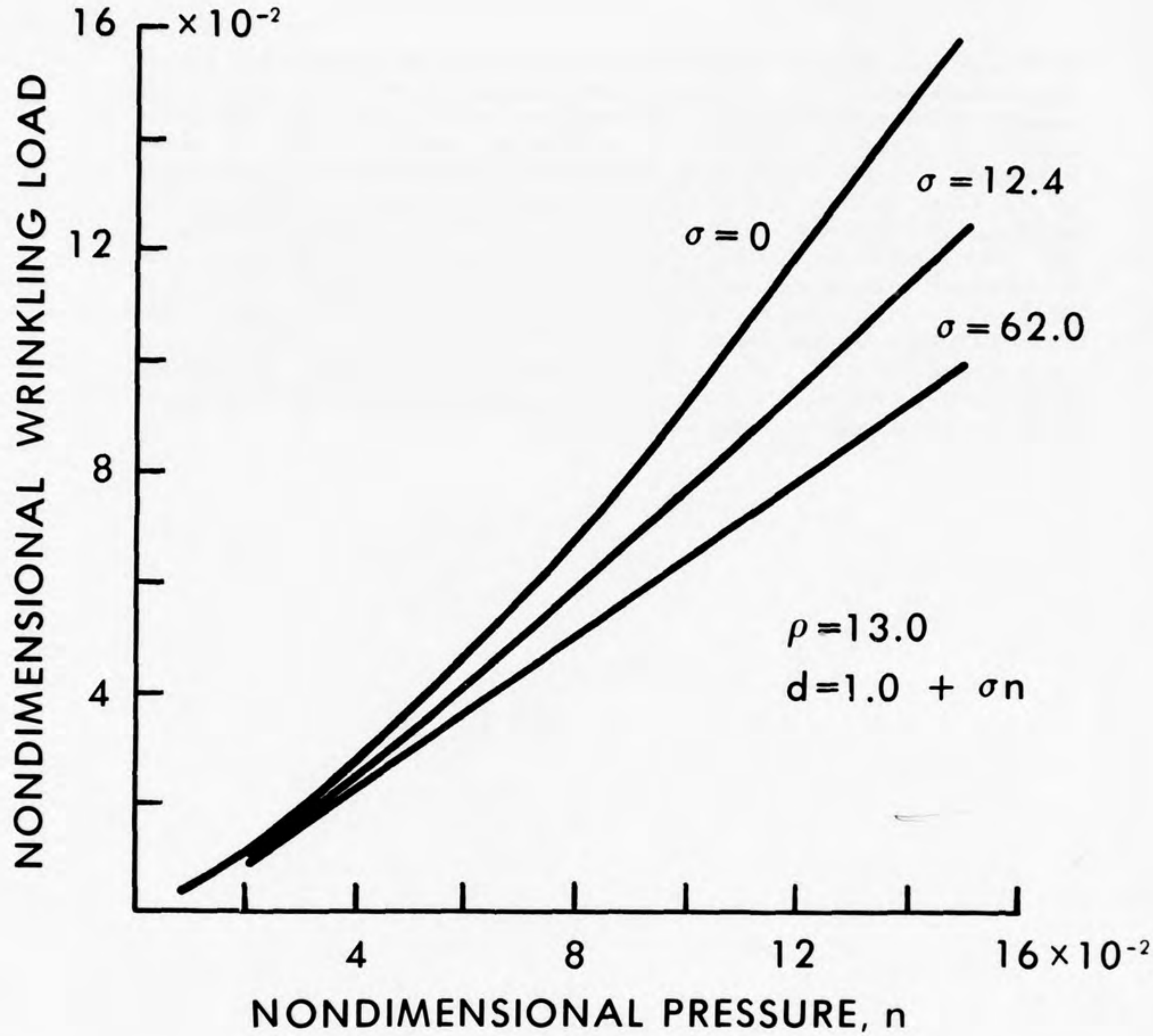


FIGURE 25. EFFECT OF MATERIAL STIFFNESS INCREASE WITH PRESSURE ON THE WRINKLING LOAD

normal load was obtained. Because of the complexity of the solutions, they are not written down explicitly but a computer program is given to carry out the solution and compute the results in terms of the deformation and stress behavior. An experimental program to measure the deformation and load-carrying capability of a series of arches is described along with measurements of elastic and shear stiffnesses of the fabric used for the arches. Results of this experimental program are presented and compared with predictions from the theory. These results are given in terms of the flexibility and wrinkling load which describe the deformation and load-carrying capability of the arches. The flexibility and wrinkling load are presented for inflation pressures ranging from 34.5 to 207.0 kPa for four arches having radius ratios of 11.3, 13.0, 19.0, and 25.0. The agreement between the experimental and theoretical results establishes the theory as being adequate for the prediction of the deformation and load-carrying capability of pressure-stabilized arches. Thus the study can provide a rational basis for the structural design of shelters using pressure-stabilized arches.

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APPENDIX A

DEVELOPMENT OF THE ONE–DIMENSIONAL ENERGY PRINCIPLE

APPENDIX A

DEVELOPMENT OF THE ONE–DIMENSIONAL ENERGY PRINCIPLE

The two-dimensional energy principle in terms of the cross-section displacements and rotation is

$$\delta\int_{\theta_2}\int_{o}^{2\pi} 1/2\Big\{C_{22}/\alpha_2^2\,[(U' - W) - \phi_y' a\cos(\theta_1) -$$

$$(\phi_z' + \phi_x)a\sin(\theta_1)]^2 + C_{33}/\alpha_2^2\,[a(\phi_x' - \phi_z) +$$

$$(W' + U + R\phi_y)\sin(\theta_1) + (V' - R\phi_z)\cos(\theta_1)]^2 +$$

$$N_{22}^o/\alpha_2^2\,[(U + W')\cos(\theta_1) - V'\sin(\theta_1) - \phi_y a\cos^2(\theta_1) - \qquad \text{(A1)}$$

$$\phi_z a\sin(\theta_1)\cos(\theta_1)]^2 - 2F_1[a\phi_x + W\sin(\theta_1) + V\cos(\theta_1)] -$$

$$2F_2[U - \phi_y a\cos(\theta_1) - \phi_z a\sin(\theta_1)] -$$

$$2F_3[V\sin(\theta_1) - W\cos(\theta_1)]\Big\}\,\alpha_2\, a\,d\theta_1\, d\theta_2$$

Expanding this expression we obtain

$$\delta\int_{\theta_2}\int_{o}^{2\pi} 1/2\Bigg\{\frac{C_{22}}{R^2(1 + {}^a/_R\cos(\theta_1))}\,[(U' - W)^2 + (a\phi_y')^2\cos^2(\theta_1) +$$

$$(a\phi_z' + a\phi_x)^2\sin^2(\theta_1) - 2(U' - W)\phi_y' a\cos(\theta_1)$$

$$-2(U' - W)(\phi_z' + \phi_x)a\sin(\theta_1) + 2\phi_y'(\phi_z' + \phi_x)a^2\sin(\theta_1)\cos(\theta_1)] +$$

$$\frac{C_{33}}{R^2(1 + {}^a/_R\cos(\theta_1))}\,[(a\phi_x' - a\phi_z)^2 + (W' + U + R\phi_y)^2\sin(\theta_1) +$$

$$(V' - R\phi_z)^2\cos^2(\theta_1) + 2(W' + U + R\phi_y)(\phi_x' - \phi_z)a\sin(\theta_1) + \qquad \text{(A2)}$$

$$2(\phi_x' - \phi_z)(V' - R\phi_z)a\cos(\theta_1) + 2(W' + U + R\phi_y)(V' - R\phi_z)\cos(\theta_1)\sin(\theta_1)] +$$

$$\frac{N^{\circ}_{22}}{R^2(1 + {}^a/_R\cos(\theta_1))} [(U + W')^2\cos^2(\theta_1) + (V')^2\sin^2(\theta_1) + (\phi_y a)^2\cos^4(\theta_1) +$$

$$(a\phi_z)^2\sin^2(\theta_1)\cos^2(\theta_1) - 2(U + W')V'\cos(\theta_1)\sin(\theta_1) -$$

$$2(U + W')\phi_y a\cos^3(\theta_1) - 2(U + W')\phi_z a\sin(\theta_1)\cos^2(\theta_1) +$$

$$2V'\phi_y a\sin(\theta_1)\cos^2(\theta_1) + 2V'\phi_z a\sin^2(\theta_1)\cos(\theta_1) +$$

$$2\phi_y\phi_z a^2\sin(\theta_1)\cos^3(\theta_1)] - 2(1 + {}^a/_R\cos(\theta_1))[UF_2 +$$

$$W(F_1\sin(\theta_1) - F_3\cos(\theta_1)) + V(F_1\cos(\theta_1) + F_3\sin(\theta_1)) +$$

$$\left.\phi_x aF_1 - \phi_y aF_2\cos(\theta_1) - \phi_z aF_2\sin(\theta_1)]\right\} aRd\theta_1 d\theta_2 = 0$$

The θ_1 dependence is all in explicit form in equation (A2), and we can thus evaluate the integrals with respect to θ_1 and this is done as follows, starting with

$$\int_o^{2\pi} \frac{({}^a/_R)^4\cos^4(\theta_1)}{1 + ({}^a/_R)\cos(\theta_1)}\, d\theta_1 = \pi(\frac{a}{R})^4 Z \tag{A3}$$

which is a definition of Z for which a formula will be given later. It should be noted that Z has a nonvanishing limit as ${}^a/_R$ becomes very small. To aid in the evaluation of the remaining integrals note that (A3) can be written with the aid of the binomial expansion as:

$$\int_o^{2\pi} \frac{({}^a/_R)^4\cos^4(\theta_1)}{1 + ({}^a/_R)\cos(\theta_1)}\, d\theta_1 =$$

(A4)

$$\int_o^{2\pi} [(\frac{a}{R})^4\cos^4(\theta_1) - (\frac{a}{R})^5\cos^5(\theta_1) + (\frac{a}{R})^6\cos^6(\theta_1) - \ldots]\, d\theta_1$$

Taking next the integral having the third power of the cosine and using the binomial expansion we obtain:

$$\int_o^{2\pi} \frac{({}^a/_R)^3\cos^3(\theta_1)\, d\theta_1}{1 + ({}^a/_R)\cos(\theta_1)} = \int_o^{2\pi} (\frac{a}{R})^3\cos^3(\theta_1)d\theta_1 - \int_o^{2\pi} [(\frac{a}{R})^4\cos^4(\theta_1) - (\frac{a}{R})^5\cos^5(\theta_1) +$$

$$(\frac{a}{R})^6\cos^6(\theta_1) - \ldots]\, d\theta_1 = -\pi(\frac{a}{R})^4 Z \tag{A5}$$

Using this process and the trigonometric identities, the remaining integrals can be evaluated as follows:

$$\int_o^{2\pi} \frac{(^a/_R)^2 \cos^2(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} d\theta_1 = \pi(\frac{a}{R})^2 [1 + (\frac{a}{R})^2 Z] = \pi(\frac{a}{R})^2 Z_2 \qquad \text{(A6)}$$

$$\int_o^{2\pi} \frac{(^a/_R)\cos(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} d\theta_1 = -\pi(\frac{a}{R})^2 Z_2 \qquad \text{(A7)}$$

$$\int_o^{2\pi} \frac{1}{1 + (^a/_R)\cos(\theta_1)} d\theta_1 = \pi[2 + (\frac{a}{R})^2 [1 + (\frac{a}{R})^2 Z]] \qquad \text{(A8)}$$

$$\int_o^{2\pi} \frac{(^a/_R)^2 \sin^2(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} d\theta_1 = \pi(\frac{a}{R})^2 [1 + (\frac{a}{R})^2 (1 - Z) + (\frac{a}{R})^4 Z] = \pi(\frac{a}{R})^2 Z_3 \qquad \text{(A9)}$$

$$\int_o^{2\pi} \frac{(^a/_R)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} d\theta_1 = 0 \qquad \text{(A10)}$$

$$\int_o^{2\pi} \frac{(^a/_R)\cos(\theta_1)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} d\theta_1 = 0 \qquad \text{(A11)}$$

$$\int_o^{2\pi} \frac{(^a/_R)\sin^2(\theta_1)\cos^2(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} d\theta_1 = \pi(\frac{a}{R})^4 [1 + Z((\frac{a}{R})^2 - 1)] = \pi(\frac{a}{R})^4 Z_4 \qquad \text{(A12)}$$

$$\int_o^{2\pi} \frac{(^a/_R)^3 \sin^2(\theta_1)\cos(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} d\theta_1 = -\pi(\frac{a}{R})^4 Z_4 \qquad \text{(A13)}$$

Thus all the integrals can be evaluated in terms of the parameter Z, and this parameter can be evaluated by the direct integration of (A8) as:

$$\int_0^{2\pi} \frac{1}{1 + (^a/_R)\cos(\theta_1)}\, d\theta_1 = 2\pi[1 - (\frac{a}{R})^2]^{-1/2} \tag{A14}$$

Equating this result with that of (A8) gives

$$Z = (\frac{R}{a})^4 [2[1 - (\frac{a}{R})^2]^{-1/2} - 2 - (\frac{a}{R})^2] \tag{A15}$$

Using these integral formulas in (A2) and rearranging, we obtain the one-dimensional energy principle

$$\delta \int \pi/2 \left\{ \frac{C_{22}}{R^2} \;[2(U' - W)^2 + Z_2(\frac{a}{R})^2(U' - W + R\phi_y')^2 + a^2 Z_3(\phi_z' + \phi_x)^2] + \right.$$

$$\frac{C_{33}}{R^2}\;[2a^2(\phi_x' - \phi_z)^2 + Z_2(\frac{a}{R})^2[a(\phi_x' - \phi_z) - \frac{R}{a}(V' - R\phi_z)]^2 +$$

$$Z_3(W' + U - R\phi_y)^2] + \frac{N^\circ_{22}}{R^2}\;[(U + W') +$$

$$Z(\frac{a}{R})^2(U + W' + R\phi_y)^2 + (V')^2 + \tag{A16}$$

$$Z_4(\frac{a}{R})^2(V' - R\phi_z)^2] - 2[Uf_2 + Wf_1 +$$

$$\left. Vf_3 + a\phi_x f_4 - a\phi_y f_5 - a\phi_z f_6] \right\} aRd\theta_2 = 0$$

The generalized force components associated with the cross-section displacement and rotations are defined by

$$f_1 = \frac{1}{\pi}\int_0^{2\pi} (F_1\sin(\theta_1) - F_3\cos(\theta_1))(1 + (^a/_R)\cos(\theta_1))d\theta_1 \tag{A17}$$

$$f_2 = \frac{1}{\pi} \int_0^{2\pi} F_2(1 + (^a/_R)\cos(\theta_1)d\theta_1 \tag{A18}$$

$$f_3 = \frac{1}{\pi} \int_0^{2\pi} (F_1\cos(\theta_1) + F_3\sin(\theta_1))(1 + (^a/_R)\cos(\theta_1))d\theta_1 \tag{A19}$$

$$f_4 = \frac{1}{\pi} \int_0^{2\pi} F_1(1 + (^a/_R)\cos(\theta_1))d\theta_1 \tag{A20}$$

$$f_5 = \frac{1}{\pi} \int_0^{2\pi} F_2\cos(\theta_1)(1 + (^a/_R)\cos(\theta_1))d\theta_1 \tag{A21}$$

$$f_6 = \frac{1}{\pi} \int_0^{2\pi} F_2\sin(\theta_1)(1 + (^a/_R)\cos(\theta_1))d\theta_1 \tag{A22}$$

This completes the detail development of the one-dimensional energy principle.

APPENDIX B

DERIVATION OF ONE–DIMENSIONAL STRESS RESULTANTS AND MOMENTS

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DERIVATION OF ONE–DIMENSIONAL STRESS RESULTANTS AND MOMENTS

The one-dimensional stress resultants and moments are defined as integrals of N'_{22} and N'_{12} over the cross-section and are expressed in terms of the cross-section displacement and rotations by use of the stress/strain law and the strain displacement relations which are repeated here for convenience.

$$N'_{22} = C_{22}\epsilon'_{22} \tag{B1}$$

$$N'_{12} = 2C_{33}\epsilon'_{12} \tag{B2}$$

$$\epsilon'_{22} = \frac{1}{R(1 + (^a/_R)\cos(\theta_1)} [(U' - W) - \phi'_y a\cos(\theta_1) - (\phi'_z + \phi_x) a\sin(\theta_1)] \tag{B3}$$

$$\epsilon'_{12} = \frac{1}{2R(1 + (^a/_R))\cos(\theta_1)} [a(\phi'_x - \phi_z) + (W' + U + R\phi_y)\sin(\theta_1) + (V' - R\phi_z)\cos(\theta_1)] \tag{B4}$$

$$\epsilon''_{22} = \frac{1}{2R^2(1 + (^a/_R)\cos(\theta_1))^2} [(U + W')\cos(\theta_1) - V'\sin(\theta_1) - \phi_y a\cos^2(\theta_1) - \phi_z a\sin(\theta_1)\cos(\theta_1)]^2 \tag{B5}$$

The axial stress resultant is defined as:

$$t = \int_0^{2\pi} N'_{22}\, a\, d\theta_1$$

$$t = \int_0^{2\pi} C_{22} \frac{a}{R} [\frac{(U' - W)}{1 + (^a/_R)\cos(\theta_1)} - \frac{\phi'_y a\cos(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \frac{(\phi'_z + \phi_x) a\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)}] d\theta_1$$

Using (A5) through (A13) for the evaluation of the integrals we have

$$t = \pi C_{22} \frac{a}{R} [(2 + (\frac{a}{R})^2 Z_2)(U' - W) + (\frac{a}{R})^2 Z_2 R\phi'_y] \tag{B6}$$

The moments about the x, y, and z axes, respectively, are defined as:

$$m_x = \int_o^{2\pi} N'_{12} a^2 d\theta_1$$

$$m_y = \int_o^{2\pi} N'_{22} a^2 \cos(\theta_1) d\theta_1$$

$$m_z = \int_o^{2\pi} N'_{22} a^2 \sin(\theta_1) d\theta_1$$

Substituting the stress/strain law and the strain displacement relations we have:

$$m_x = \int_o^{2\pi} C_{33} \frac{a^2}{R} [\frac{a(\phi'_x - \phi_z)}{1 + (^a/_R)\cos(\theta_1)} + \frac{(V' - R\phi_z)\cos(\theta_1)}{(1 + (^a/_R)\cos(\theta_1)} + \frac{(W' + U + R\phi_y)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)}] d\theta_1$$

$$m_y = \int_o^{2\pi} C_{22} \frac{a^2}{R} [\frac{(U' - W)\cos(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \frac{\phi'_y a\cos^2(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \frac{(\phi'_z + \phi_x) a\cos(\theta_1)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)}] d\theta_1$$

$$m_z = \int_o^{2\pi} C_{22} \frac{a^2}{R} [\frac{(U' - W)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \frac{\phi'_y a\cos(\theta_1)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \frac{(\phi'_z + \phi_x) a\sin^2(\theta_1)}{1 + (^a/_R)\cos(\theta_1)}] d\theta_1$$

Carrying out the integral with respect to θ_1 using the formulas (A5) through (A13) we have

$$m_x = \pi C_{33} \frac{a^3}{R^2} [R(2 + (\frac{a}{R})^2 Z_2)(\phi'_x + \phi_z) - Z(V' - R\phi_z)] \tag{B7}$$

$$m_y = -\pi C_{22} Z_2 \frac{a^3}{R^2} [U' - W + R\phi'_y] \tag{B8}$$

$$m_z = -\pi C_{22} Z_3 \frac{a^3}{R^2} [R(\phi'_z + \phi_x)] \tag{B9}$$

The remaining two stress resultants are the transverse shear resultant defined by:

$$q_z = \int_o^{2\pi} [N'_{12}\sin(\theta_1) + N^\circ_{22} \sqrt{2\epsilon''_{22}} \cos(\theta_1)]\, a d\theta_1$$

$$q_y = \int_o^{2\pi} [N'_{12}\cos(\theta_1) - N^\circ_{22} \sqrt{2\epsilon''_{22}} \sin(\theta_1)\, a d\theta_1$$

Substituting the stress/strain law and the strain displacement relations we have:

$$q_z = \int_o^{2\pi} \Bigg\{ C_{33} \frac{a}{R} [\frac{(\phi'_x - \phi_z) a \sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} + \frac{(V' - R\phi_z)\cos(\theta_1)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} +$$

$$\frac{(W' + U + R\phi_y)\sin^2\theta}{1 + (^a/_R)\cos(\theta_1)}] + N^\circ_{22} \frac{a}{R} [\frac{(U + W')\cos^2(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \frac{\phi_y a \cos^3(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} -$$

$$\frac{V'\sin(\theta_1)\cos(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \frac{\phi_z a \sin(\theta_1)\cos^2(\theta_1)}{1 + (^a/_R)\cos(\theta_1)}] \Bigg\} d\theta_1$$

$$q_y = \int_o^{2\pi} \left\{ C_{33} \frac{a}{R} \left[\frac{(\phi_x' - \phi_z)a\cos(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} + \frac{(W' + U + R\phi_y)\cos(\theta_1)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} + \right.\right.$$

$$\left. \frac{(V' - R\phi_z)\cos^2(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} \right] - N_{22}^{\circ} \frac{a}{R} \left[\frac{(U + W')\cos(\theta_1)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \frac{V'\sin^2(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \right.$$

$$\left.\left. \frac{\phi_y a\cos^2(\theta_1)\sin(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} - \frac{\phi_z a\sin^2(\theta_1)\cos(\theta_1)}{1 + (^a/_R)\cos(\theta_1)} \right] \right\} d\theta_1$$

Again we integrate with respect to θ_1 using (A5) through (A13) to give

$$q_z = \pi C_{33} Z_3 \frac{a}{R} (W' + U + R\phi_y) + \pi N_{22}^{\circ} \frac{a}{R} [Z_2(U + W') + (\frac{a}{R})^2 ZR\phi_y] \tag{B10}$$

$$q_y = \pi C_{33} Z_2 \frac{a}{R} [(V' - R\phi_z) - (\frac{a}{R})^2 R(\phi_x' - \phi_z)] - \pi N_{22}^{\circ} \frac{a}{R} (-Z_3 V' + (\frac{a}{R})^2 RZ_4\phi_z) \tag{B11}$$

These stress resultants and moments are shown graphically in Figure 2. In addition to being the definitions of the stress resultants and moments, the above expressions are also the force displacement relations for the one-dimensional theory.

APPENDIX C

THEORY IN NONDIMENSIONAL FORM

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THEORY IN NONDIMENSIONAL FORM

When obtaining solutions and results from theory it is useful to have the equations, including the basic definitions, in nondimensional form. To do this for the present theory, we begin by defining nondimensional displacements.

$$\mathit{U} = \mathrm{U/a} \tag{C1}$$

$$\mathit{W} = \mathrm{W/a} \tag{C2}$$

$$\mathit{V} = \mathrm{V/a} \tag{C3}$$

The nondimensional strain measures are obtained from equations (18) as

$$\mathrm{E} = (U' - W)/\rho \tag{C4}$$

$$\mathrm{K_y} = \mathrm{R}\kappa_y = \phi'_y \tag{C5}$$

$$\mathrm{K_z} = \mathrm{R}\kappa_z = \phi'_z + \phi_x \tag{C6}$$

$$\mathrm{K_x} = \mathrm{R}\kappa_x = \phi'_x - \phi_z \tag{C7}$$

$$\Gamma_z = (U + W' + \rho\phi_y)/\rho \tag{C8}$$

$$\Gamma_y = (V' - \rho\phi_z)/\rho \tag{C9}$$

where $\rho = \mathrm{R/a}$. The nondimensional stress resultant and moments are defined as follows, and the nondimensional stress-displacement relations follow from equations (16):

$$\mathrm{T} = \mathrm{t}/\bar{\mathrm{C}}_{22}\mathrm{a} = (\pi \mathrm{d}/\rho)[(2 + \mathrm{Z}_2/\rho^2)(U' - W) + \mathrm{Z}_2\phi'_y/\rho] \tag{C10}$$

$$\begin{aligned}\mathrm{Q_y} = \mathrm{q_y}/\bar{\mathrm{C}}_{22}\mathrm{a} &= (\pi \mathrm{c}\mathrm{Z}_2/\rho)[(V' - \rho\phi_z) - (\phi'_x - \phi_z)/\rho] - \\ &\quad (\pi \mathrm{n}/\rho)[-\mathrm{Z}_3 V' + \mathrm{Z}_4\phi_z/\rho]\end{aligned} \tag{C11}$$

$$\begin{aligned}\mathrm{Q_z} = \mathrm{q_z}/\bar{\mathrm{C}}_{22}\mathrm{a} &= (\pi \mathrm{c}\mathrm{Z}_3/\rho)[W' + U + \rho\phi_y] + \\ &\quad (\pi \mathrm{n}/\rho)[\mathrm{Z}_2(U + W') + \mathrm{Z}\phi_y/\rho]\end{aligned} \tag{C12}$$

$$\mathrm{M_x} = \mathrm{m_x}/\bar{\mathrm{C}}_{22}\mathrm{a}^2 = (\pi \mathrm{c}/\rho)[(2 + \mathrm{Z}_2/\rho^2)(\phi'_x - \phi_z) - \mathrm{Z}_2(V' - \rho\phi_z)/\rho] \tag{C13}$$

$$M_y = m_y/\bar{C}_{22}a^2 = -(\pi d Z_2/\rho^2)[U' - W + \rho\phi_y'] \tag{C14}$$

$$M_z = m_z/\bar{C}_{22}a^2 = -(\pi d Z_3/\rho^2)[\phi_z' + \phi_x] \tag{C15}$$

The equilibrium equations in terms of the nondimensional stress resultants and moments are obtained by dividing equations (20) by $C_{22}a$ to yield

$$-T' + Q_z - \pi\bar{f}_2 = 0 \tag{C16}$$

$$-(T + Q_z') - \pi\bar{f}_1 = 0 \tag{C17}$$

$$M_y'/\rho + Q_z - (\pi n/\rho)(U + W') + \pi\bar{f}_5/\rho = 0 \tag{C18}$$

$$-Q_y' - \pi\bar{f}_3 = 0 \tag{C19}$$

$$(M_z' - M_x)/\rho - Q_y + (\pi n/\rho)V' + \pi\bar{f}_6/\rho = 0 \tag{C20}$$

$$-(M_z + M_x')/\rho - \pi\bar{f}_4/\rho = 0 \tag{C21}$$

The nondimensional forces are given as $\bar{f}_i = Rf_i/C_{22}$ and $n = N^\circ_{22}/\bar{C}_{22}$ is the nondimensional stress due to the internal pressure. The parameters $d = C_{22}/\bar{C}_{22}$ and $c = C_{33}/\bar{C}_{22}$ are the nondimensional stiffnesses and $\bar{C}_{22}$ is a reference value of C_{22}. The governing equations in terms of the nondimensional displacement parameters are obtained from equations (22) by dividing by $\bar{C}_{22}a$ to give

$$\begin{aligned}&-[2d/\rho + dZ_2/\rho^3]\, U'' + [cZ_3/\rho + nZ_2/\rho]U - [dZ_2/\rho^2]\phi_y'' + \\ &[cZ_3 + nZ/\rho^2]\phi_y + [2d/\rho + dZ_2/\rho^3 + cZ_3/\rho + nZ_2/\rho]W' - \bar{f}_2 = 0\end{aligned} \tag{C22}$$

$$\begin{aligned}&-[cZ_3/\rho + nZ_2/\rho]W'' + [d(2 + Z_2/\rho^2)/\rho]W - [dZ_2/\rho^2 + cZ_3 + nZ/\rho^2]\phi_y' - \\ &[d(2 + Z_2/\rho^2)/\rho + cZ_3/\rho + nZ_2/\rho]U' - \bar{f}_1 = 0\end{aligned} \tag{C23}$$

$$\begin{aligned}&-[dZ_2/\rho]\phi_y'' + [cZ_3\rho + nZ/\rho]\phi_y - [dZ_2/\rho^2]U'' + [cZ_3 + nZ/\rho^2]U + \\ &[dZ_2/\rho^2 + cZ_3 + nZ/\rho^2]W' + \bar{f}_5 = 0\end{aligned} \tag{C24}$$

$$\begin{aligned}&-[cZ_2/\rho + nZ_3/\rho]V'' + [cZ_2/\rho^2]\phi_x'' + \\ &[cZ_2(1 - 1/\rho^2) + nZ_4/\rho^2]\phi'_z - \bar{f}_3 = 0\end{aligned} \tag{C25}$$

$$\begin{aligned}&-[dZ_3/\rho^2]\phi_z'' + [c(2/\rho^2 + Z_2(1 - 1/\rho^2)^2] + nZ_4/\rho^2]\phi_z - \\ &[dZ_3/\rho^2 + c(2 - Z_2(1 - 1/\rho^2))/\rho^2]\phi_x' - \\ &[cZ_2(1 - 1/\rho^2)/\rho - n(1 - Z_3)/\rho]V' + \bar{f}_6/\rho = 0\end{aligned} \tag{C26}$$

$$-[c(2 + Z_2/\rho^2)/\rho^2]\phi_X'' + [dZ_3/\rho^2]\phi_X + [cZ_2/\rho^3]V'' +$$
$$[dZ_3/\rho^2 + c[2 - Z_2(1 - 1/\rho^2))/\rho^2]\phi_Z' - \bar{f}_4/\rho = 0 \quad (C27)$$

The boundary conditions are similarly expressed in terms of the nondimensional displacement parameter or the nondimensional stress resultants and moments. The Green's function discontinuity conditions are put in nondimensional form by dividing the stress resultant equations by $a\bar{C}_{22}$ and the moment equations by $a^2\bar{C}_{22}$ to give

$$T_1 - T_2 = \pi\bar{g}_2 \quad (C28)$$

$$M_{y1} - M_{y2} = \pi\bar{g}_5 \quad (C29)$$

$$Q_{z1} - Q_{z2} = \pi\bar{g}_1 \quad (C30)$$

$$M_{z1} - M_{z2} = \pi\bar{g}_6 \quad (C31)$$

$$M_{x1} - M_{x2} = \pi\bar{g}_4 \quad (C32)$$

$$Q_{y1} - Q_{y2} = \pi\bar{g}_3 \quad (C33)$$

where the nondimensional forces are $\bar{g}_i = g_i/C_{22}$, $i = 1,6$.

APPENDIX D

RELATIONSHIP AMONG THE CONSTANTS OF INTEGRATION

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RELATIONSHIP AMONG THE CONSTANTS OF INTEGRATION

The homogenous solution as expressed by equations (37) contains 18 unknown constants. But, because the characteristic numbers cause the determinate of equation (33) to vanish, there is for each of these characteristic numbers a relationship among these constants which is used here to express the A_i and B_i in terms of the C_i. To obtain these relationships the solution (37) is substituted into the governing differential equations (30) and causing the coefficients of the functions $\cosh(\lambda\theta_2)$, $\sinh(\lambda\theta_2)$, $\cos(\theta_2)$, $\sin(\theta_2)$, $\theta_2\cos(\theta_2)$ and $\theta_2\sin(\theta_2)$ to vanish independently yielding 18 homogenous equations in 18 unknowns. This set of equations, however, has a rank of only 12, thus 12 of the unknowns can be expressed in terms of the remaining 6. The 18 equations in matrix form take the form shown on the following page

where

$$
\begin{aligned}
J_1 &= H_2 - \lambda^2 H_1 \\
J_2 &= H_4 - \lambda^2 H_3 \\
J_3 &= H_1 + H_2 \\
J_4 &= H_3 + H_4 \\
J_5 &= H_1 - \lambda^2 H_2
\end{aligned}
\tag{2D}
$$

Examination of these equations reveals that the relation among the constants A_1, B_1 and C_1 is the same as that among A_2, B_2, and C_2; thus we obtain from two of the three equations

$$
\begin{Bmatrix} A_1 \\ \\ B_1 \end{Bmatrix} = \begin{bmatrix} \beta_1 \\ \\ \beta_2 \end{bmatrix} \begin{Bmatrix} C_2 \end{Bmatrix}
$$

and

$$
\begin{Bmatrix} A_2 \\ \\ B_2 \end{Bmatrix} = \begin{bmatrix} \beta_1 \\ \\ \beta_2 \end{bmatrix} \begin{Bmatrix} C_1 \end{Bmatrix}
$$

$$
\begin{bmatrix}
J_1 & J_2 & \lambda J_3 &&&&&&&&&&&&&&& \\
J_2 & \rho J_2 & \lambda J_4 &&&&&&&&&&&&&&& \\
-\lambda J_3 & -\lambda J_4 & J_5 &&&&&&&&&&&&&&& \\
&&& J_1 & J_2 & \lambda J_3 &&&&&&&&&&&& \\
&&& J_2 & \rho J_2 & \lambda J_4 &&&&&&&&&&&& \\
&&& -\lambda J_3 & -\lambda J_4 & J_5 &&&&&&&&&&&& \\
&&&&&& J_3 & J_4 & 2H_1 & 2H_3 & -J_3 & J_3 &&&&&& \\
&&&&&& J_4 & \rho J_4 & 2H_3 & 2\rho H_3 & -J_4 & J_4 &&&&&& \\
&&&&&& -J_3 & -J_4 & -J_3 & -J_4 & J_3 & -2H_2 &&&&&& \\
&&&&&&&& J_3 & J_4 && J_3 &&&&&& \\
&&&&&&&& J_4 & \rho J_4 && J_4 &&&&&& \\
&&&&&&&& J_3 & J_4 && J_3 &&&&&& \\
&&&&&&&&&&&& J_3 & J_4 & 2H_1 & 2H_3 & -J_3 & J_3 \\
&&&&&&&&&&&& J_4 & \rho J_4 & 2H_3 & 2\rho H_3 & -J_4 & J_4 \\
&&&&&&&&&&&& -J_3 & -J_4 & -J_3 & -J_4 & J_3 & -2H_2 \\
&&&&&&&&&&&&&& J_3 & J_4 && J_3 \\
&&&&&&&&&&&&&& J_4 & \rho J_4 && J_4 \\
&&&&&&&&&&&&&& J_3 & J_4 && J_3
\end{bmatrix}
\begin{Bmatrix}
A_1 \\ B_1 \\ C_2 \\ A_2 \\ B_2 \\ C_1 \\ A_3 \\ B_3 \\ A_6 \\ B_6 \\ C_4 \\ C_5 \\ A_4 \\ B_4 \\ -A_5 \\ -B_5 \\ -C_3 \\ C_6
\end{Bmatrix}
= 0 \tag{1D}
$$

where

$$\begin{bmatrix} \beta_1 \\ \\ \beta_2 \end{bmatrix} = - \begin{bmatrix} J_1 & J_2 \\ \\ J_2 & \rho J_2 \end{bmatrix}^{-1} \begin{bmatrix} \lambda J_3 \\ \\ \lambda J_4 \end{bmatrix}$$

Similarly, the relation among A_3, B_3, A_6, B_6, C_4, and C_5 is the same as that among A_4, B_4, $-A_5$, $-B_5$, $-C_3$, and C_6; thus we obtain from equations (7), (8), (10), and (11)

$$\begin{Bmatrix} A_3 \\ B_3 \\ A_6 \\ B_6 \end{Bmatrix} = \begin{bmatrix} \beta_3 & \beta_5 \\ \beta_4 & \beta_6 \\ 0 & -\beta_3 \\ 0 & -\beta_4 \end{bmatrix} \begin{Bmatrix} C_4 \\ C_5 \end{Bmatrix}$$

and

$$\begin{Bmatrix} A_4 \\ B_4 \\ -A_5 \\ -B_5 \end{Bmatrix} = \begin{bmatrix} \beta_3 & \beta_5 \\ \beta_4 & \beta_6 \\ 0 & -\beta_3 \\ 0 & -\beta_4 \end{bmatrix} \begin{Bmatrix} -C_3 \\ C_6 \end{Bmatrix}$$

where

$$\begin{bmatrix} \beta_3 & \beta_5 \\ \beta_4 & \beta_6 \\ 0 & -\beta_3 \\ 0 & -\beta_4 \end{bmatrix} = \begin{bmatrix} J_3 & J_4 & 2H_1 & 2H_3 \\ J_4 & \rho J_4 & 2H_3 & 2\rho H_3 \\ 0 & 0 & J_3 & J_4 \\ 0 & 0 & J_4 & \rho J_4 \end{bmatrix}^{-1} \begin{bmatrix} -J_3 & J_3 \\ -J_4 & J_4 \\ 0 & J_3 \\ 0 & J_4 \end{bmatrix}$$

APPENDIX E

SYSTEMS OF EQUATIONS FOR DETERMINATION OF THE CONSTANTS OF INTEGRATION

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SYSTEMS OF EQUATIONS FOR DETERMINATION OF THE CONSTANTS OF INTEGRATION

Our purpose here is to give the expressions for the coefficients of the systems of equations which determine the constants of integration for the solution of the arch equations. The coefficients for the Green's function will be given first, followed by those for the uniform load solution.

The Green's Function contains 12 constants of integration; therefore, 12 equations are required for their determination. Six of these constants, C_1 through C_6, define the solution over $-\alpha < \theta_2 \leqslant \bar{\theta}_2$ and the remaining six define the solution over $\bar{\theta}_2 \leqslant \theta_2 \leqslant \alpha$. These equations can be written in matrix form as

$$[S]\ C = b$$

where C is the vector of 12 constants of integration and the matrix of coefficients S are presented below. The first three equations impose the boundary conditions at $\theta_2 = -\alpha$, and equations (4) through (6) impose the boundary conditions at $\theta_2 = \alpha$. Coefficients will be given for both the simply supported and the fixed boundary conditions. Equations (7) through (9) state the continuity of displacements and rotation at $\theta_2 = \bar{\theta}_2$, the location of the concentrated load and equations (10) through (12) are the discontinuity condition on the axial force, the moment and the transverse shear force. The coefficients, $s_{ij}(i = 1,3;\ j = 1,6)$ are evaluated with ease by evaluation of U_1, W_1, and ϕ_{y_1} at $\theta_2 = -\alpha$ for the fixed boundary condition and U_1, W_1, and $(U'_1 - W_1 + \rho\phi'_{y_1})$ at $\theta_2 = -\alpha$ for the simply supported boundary conditions. For s_{ij} $(i = 4,6;\ j = 7,12)$ the parameters U_2, W_2, and ϕ_{y_2} replace U_1, W_1, and ϕ_{y_1} and are evaluated at $\theta_2 = \alpha$. In addition, we have $s_{ij} = 0$ for $i = 1,3;\ j = 7,12$ and $i = 4,6;\ j = 1,6$. Since equations (7) through (9) impose continuity of displacement at $\theta_2 = \bar{\theta}_2$ the coefficients, $s_{ij}((i = 7,9;\ j = 1,6,)$ are found by evaluating U_1, W_1, and ϕ_{y_1} at $\theta_2 = \bar{\theta}_2$ and the coefficients, s_{ij} $(i = 7,9;\ j = 7,12,)$ are found by evaluating U_2, W_2, and ϕ_{y_2} at $\theta_2 = \bar{\theta}_2$. The coefficients of the last three equations are given in detail below.

$$s_{10,1} = L_1 \cosh(\lambda\bar{\theta}_2)$$

$$s_{10,2} = L_1 \sinh(\lambda\bar{\theta}_2)$$

$$s_{10,3} = L_2 \sin(\bar{\theta}_2)$$

$$s_{10,4} = L_2 \cos(\bar{\theta}_2)$$

$$s_{10,5} = L_3 \cos(\bar{\theta}_2) + L_2\bar{\theta}_2 \sin(\bar{\theta}_2)$$

$$s_{10,6} = -L_3 \sin(\bar{\theta}_2) + L_2 \bar{\theta}_2 \cos(\bar{\theta}_2)$$

$$L_1 = (\beta_1 \lambda - 1) H_1 + H_3 \beta_2 \lambda$$

$$L_2 = (\beta_3 - 1) H_1 + H_3 \beta_4$$

$$L_3 = (\beta_5 - \beta_3) H_1 + (\beta_6 - \beta_4) H_3$$

$$s_{11,1} = L_4 \cosh(\lambda \bar{\theta}_2)$$

$$s_{11,2} = L_4 \sinh(\lambda \bar{\theta}_2)$$

$$s_{11,3} = L_5 \sin(\bar{\theta}_2)$$

$$s_{11,4} = L_5 \cos(\bar{\theta}_2)$$

$$s_{11,5} = L_6 \cos(\bar{\theta}_2) + L_5 \bar{\theta}_2 \sin(\bar{\theta}_2)$$

$$s_{11,6} = -L_6 \sin(\bar{\theta}_2) + L_5 \theta_2 \cos(\bar{\theta}_2)$$

$$L_4 = [(\beta_1 \lambda - 1) + \rho \beta_2 \lambda] H_3$$

$$L_5 = [(\beta_3 - 1) + \rho \beta_4] H_3$$

$$L_6 = [(\beta_5 - \beta_3) + \rho(\beta_6 - \beta_4)] H_3$$

$$s_{12,1} = L_7 \sinh(\lambda \bar{\theta}_2)$$

$$s_{12,2} = L_7 \cosh(\lambda \bar{\theta}_2)$$

$$s_{12,3} = L_8 \cos(\bar{\theta}_2)$$

$$s_{12,4} = -L_8 \sin(\bar{\theta}_2)$$

$$s_{12,5} = L_9 \sin(\bar{\theta}_2) + L_8 \bar{\theta}_2 \cos(\bar{\theta}_2)$$

$$s_{12,6} = L_9 \cos(\bar{\theta}_2) - L_8 \bar{\theta}_2 \sin(\bar{\theta}_2)$$

$$L_7 = (\lambda + \beta_1) H_2 + H_4 \beta_2$$

$$L_8 = (1 - \beta_3) H_2 - H_4 \beta_4$$

$$L_9 = (1 + \beta_5) H_2 + H_4 \beta_6$$

The last six colums of these three equations are given by the relation

$$s_{i,\, j\, +\, 6} = -s_{i,j}$$

$$i = 10,12$$
$$j = 1,6$$

The system of equations is completed by specification of the elements of the force matrix, b, as follows

$$b_i = 0 \qquad i = 1,9$$

$$b_{10} = \bar{g}_2$$

$$b_{11} = -\bar{g}_5$$

$$b_{12} = -\bar{g}_1$$

where $\bar{g}_1$, $\bar{g}_2$, $\bar{g}_5$ are the nondimensional applied forces.

For the uniform load, the symmetry of the structure and the load about $\theta_2 = 0$ makes it possible to reduce the number of constants of integration to three, which are determined from the boundary conditions. The equation for these three constants can be written in matrix form as:

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \begin{Bmatrix} C_1 \\ C_4 \\ C_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \rho f_1/d(2 + Z_2/\rho^2) \end{Bmatrix}$$

For fixed boundary conditions the coefficients are

$$s_{11} = \beta_1 \sinh(\lambda\alpha)$$

$$s_{12} = \beta_3 \sin(\alpha)$$

$$s_{13} = \beta_5 \sin(\alpha) - \beta_3 \alpha\cos(\alpha)$$

$$s_{21} = \beta_2 \sinh(\lambda\alpha)$$

$$s_{22} = \beta_4 \sin(\alpha)$$

$$s_{23} = \beta_6 \sin(\alpha) - \beta_4 \alpha\cos(\alpha)$$

$$s_{31} = \cosh(\lambda\alpha)$$

$$s_{32} = \cos(\alpha)$$

$$s_{33} = \alpha\sin(\alpha)$$

and for simply supported boundary conditions, the second equation is changed as follows to set the moment equal to zero.

$$s_{21} = [(\beta_1\lambda - 1)H_1 + H_3\beta_2\lambda]\cosh(\lambda\alpha)$$

$$s_{22} = [(\beta_3 - 1)H_1 + H_3\beta_4]\cos(\alpha)$$

$$s_{23} = [(\beta_5 - \beta_3)H_1 + (\beta_6 - \beta_4)H_3]\cos(\alpha) +$$

$$[(\beta_3 - 1)H_1 + H_3\beta_4]\alpha\sin(\alpha)$$

APPENDIX F

COMPUTER PROGRAM FOR SOLUTION OF GOVERNING EQUATIONS

APPENDIX F

COMPUTER PROGRAM FOR SOLUTION OF GOVERNING EQUATIONS

Our purpose here is to describe the computer program which carries out the computations involved to obtain specific results from the solution described in the body of the report and Appendix E.

General Organization of Program

The flow of calculations in the program, a copy of which is given at the end of the appendix, is illustrated by the flow chart in Figure F–1. This flow chart is not intended to depict the details of the calculation but only the general organization or sequence of the calculations. The program includes a main program which calls 2 subroutines peculiar to this program, all of which carry out computations in double precision. The program also calls a system subroutine for the solution of system of linear equation. Because the program is in double precision this system subroutine must also be double precision. The program was written for the UNIVAC 1106 computer with the EXEC–8 operating system and the system subroutine called is associated with this computer system. It is believed that the program written in FORTRAN could be easily adapted to other systems.

The program contains options for the type of loading, the boundary conditions and plotting the output. The loading option includes a uniform normal loaded, a concentrated load either normal to the arch or vertical and at an arbitrary position, and a general distributed load. The concentrated load solution is a Green's function which is used to compute the solution for the general distributed load by quadrature. The functional form of the distributed load must be specified in SUBROUTINE CALPAR. The quadrature for the general load is carried out using the Gaussian numerical technique. The coordinates and weighting factors for this technique must be on logical unit 10 and written according to FORMAT (2E18.8). Two boundary condition options are provided. One has both ends fixed and the other both ends simply supported. Other options could be included by modification of the program. The plotting option when exercised provides for the plotting of the computed flexibility and wrinkling load as a function of the pressure parameter. In addition, experimental results will be plotted along with the corresponding calculated results. This experimental data must be located on logical unit 16 and written according to FORMAT (5E15.6). The CALCOMP plotter is used, and it operates a plot tape which must be assigned as logical unit 20. The program also uses a scratch file which must be assigned as logical unit 15.

Input and Execution

The input, both control parameters and data, required to operate the program is described in this section.

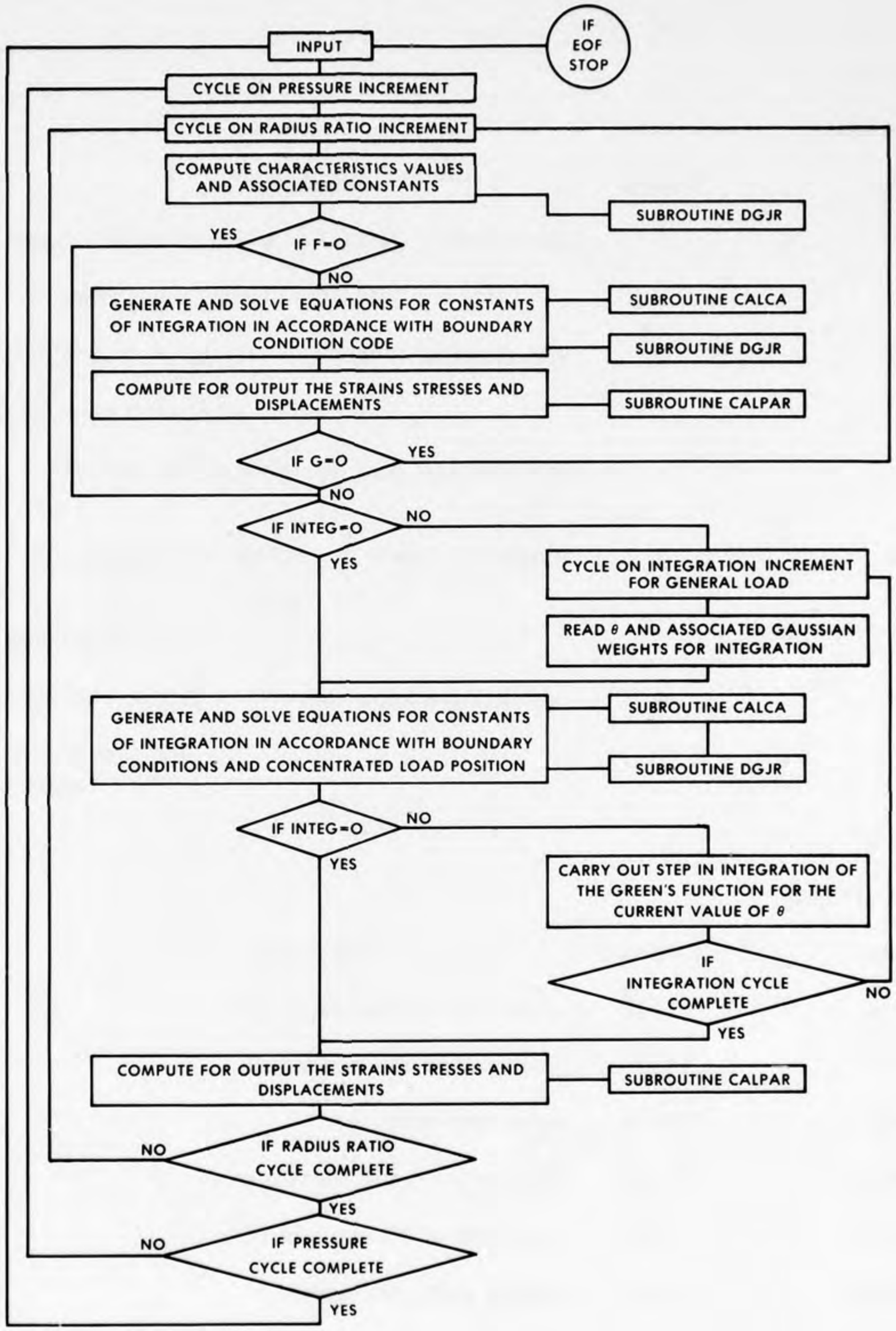

FIGURE F-1 FLOW CHART OF ARCH ANALYSIS PROGRAM

CARD #1

Format (I4)

Parameter	Column	Description
IPLOT	1	plot control IPLOT = 1 output is not plotted IPLOT = 0 output is plotted
ILOAD	2	load direction control ILOAD = 1 vertical load ILOAD = 0 normal load this parameter does not apply to the uniform load
INTEG	3	Integration control INTEG = 1 general load INTEG = 0 uniform and concentrated load
IBOUN	4	boundary condition control IBOUN = 0 fixed IBOUN = 1 simply supported

CARD #2

Format (7F10.0)

Parameter	Column	Description
RLIM(1)	1–10	initial value of the radius ratio
RLIM(2)	11–20	final value of the radius ratio
RLIM(3)	21–30	radius ratio increment
XNLIM(1)	31–40	initial value of the pressure parameter
XNLIM(2)	41–50	final value of the pressure parameter
XNLIM(3)	51–60	pressure parameter increment

CARD #3

Format (3F10.0)

Parameter	Column	Description
ALPHA	1–10	half span angle of the arch
THETA	11–20	location of concentrated load on the arch
THEHAT	21–30	location of load on the cross-section, see equation (43)

CARD #4

Format (2F10.0, I5)

Parameter	Column	Description
F	1–10	intensity of uniform load; F = 0 causes the uniform load calculation to be omitted
SMG	11–20	concentrated load magnitude; SMG = 0 causes the concentrated load computation to be omitted. For the general distributed load SMG must be put equal to unity
NMAX	21–25	number of output intervals; if NMAX = 18 and the arch spans 180°, program will print output every 10°.

CARD #5

Format (7F10.7)

Parameter	Column	Description
AC	1–10	the program allows the nondimensional shear
BC	11–20	modulus, c, and the nondimensional elastic
AE	21–30	modulus, d, to be linear functions of the
BE	31–40	pressure parameter, XN. These four parameters

define these linear functions as

c = AC*XN + BC

d = AE*XN + BE

CARD #6

Format (IX, 13A6)

Parameter	Column	Description
QIDEN	2–79	an identification of the job to be printed out with the output

An example of the coded input is given in Table F–1. This input was used to generate the sample output to be discussed subsequently. The execution of the program on the UNIVAC 1106 computer with the EXEC–8 operating system can be accomplished with the runstream shown on page 107. This includes reduction of experimental data for plotting, computation of Gaussian integration constants, assignment of the required files, compilation and assembly of the arch analysis program, execution of the program and input data. The computer output resulting from the execution of this runstream is present on pages 108 to 144. The runstream begins with the assignment of tape no. 1962 which contains the raw experimental data presented in this report and a program entitled NONDIM which reads the raw data from file 15, reduces this data to nondimensional form, and writes the nondimensional data on file 16. If it is desired to plot different flexibility and wrinkling load data against pressure, the raw data must be entered into file 15 before execution of the data reduction program. If no plotting is desired, statements 1–16 of the runstream may be omitted. Upon execution of the data reduction program, the reference value of the elastic modulus is called for as input. This input is the number 313.0 appearing as statement 12 in the runstream. This is the English unit system equivalent of the number given in the body of the report and is given in these units because the raw data is in English units. This number must also be changes for a different set of data. If plots are to be made, this experimental data must be present. Following the data reduction we have the generation of the Gaussian integration constants. This is accomplished with a short FORTRAN program which calls the subroutine LGAUSS to generate the constants, and upon return, writes them on file 10. As presented, 90 integration increments are used. A change in the number of increments is accomplished by changing the parameter N on line 20 of the runstream. The subroutine LGAUSS computes the integration coordinates over the range −1 to 1. These coordinates are transformed to the range $-\alpha$ to α at the time they are used in the arch analysis. If either a concentrated load or a uniform load is being treated which does not require integration, then statements 17 through 32 may be omitted from the runstream. After generation of the integration constants tape no. 1964 is called and the arch analysis programs are read into a mass storage file. These programs are compiled and collected using the MAP processor. The absolute executable program is put in B. If a general distributed load is being treated, its functional form must be specified in subroutine CALPAR which is the symbolic element TPF$.ARCHRESULTS. This must be done before compilation of this subroutine. If plots are to be made, a plot tape assigned as file 20 must be assigned before execution of the program.

Table F1. Input data in coded format.

Cols 1–10	Cols 11–20	Cols 21–30	Cols 31–40	Cols 41–50	Cols 51–60
1001					
20.0	30.0	10.0	0.05667	0.06667	0.01
90.0	0.0	0.0			
0.0	-0.0000663	10			
0.0	0.05	0.0	1.0		
FINITE ELEMENT CHECK CASE					

Output

A sample of the output generated by the program is given in pages 141 to 144. This output corresponds to the input given in Table F1. As is seen, the heading contains a title followed by specification of the type of loading and the boundary conditions. The user-provided-problem description, the linear expressions for the shear and elastic moduli and the problem parameters then appear. The main body of the output consists of the normal and longitudinal deflection, the rotation, the strains at the outer and inner arch radii, the axial force, and the moment for a series of equal spaced angular positions. As can be seen, these positions begin with the smallest and increase to the largest position. All of the output are in nondimensional form.

```
ECS*JUNK(1).RSAREP
        @ASG,T    TP.,6C9,1962  .  TAPE WITH DATA & REDUCTION PROGRAM
        @MOVE     TP,2
        @COPIN    TP.,
        @FREE     TP.
        @ASG,T    15.,F14   .  SCRATCH FILE
        @ASG,T    16.,F14  .  FILE WITH EXPERIMENTAL DATA FOR PLOTTING
        @DATA,IL  15.
        @ADD,D    TPF$.RAWADA
        @END
        @FOR,S    TPF$.NONDIM,N
        @XQT
        313.0
        @DATA,L   16.
        @END
        @ERS
        @ERS      15.
        @ASG,T    10.,F14    .  FILE WITH GAUSSIAN INTEGRATION CONSTANTS
        @FOR,IS
                DIMENSION X(100),W(100)
                N=90
                CALL LGAUSS(X,W,N,$1)
          1     CONTINUE
                WRITE(10,2) N
          2     FORMAT(I4)
                WRITE(10,3) (X(I),W(I),I=1,N)
          3     FORMAT(2E18.8)
                STOP
                END
        @XQT
        @DATA,L   10.
        @END
        @ERS
        @ASG,T    TP.,6C9,1964  .  TAPE WITH ARCH ANALYSIS PROGRAMS
        @MOVE     TP,7
        @COPIN    TP.,
        @FREE     TP.
        @FOR,S    TPF$.MAINARCH,M
        @FOR,S    TPF$.ARCHRESULTS,R
        @FOR,S    TPF$.BOUNDARYARCH,B
        @MAP,IN   A,B
            IN   TPF$.M
          LIB   CALCOMP*BASIC.
          END
        @ASG,T    20.,6C9,0928W  .  PLOT TAPE WITH APPROPRIATE NO.
        @XQT   B
        1001
        20.0       30.0       10.0       0.05667   0.06667   0.01
        90.0       0.0        0.0
        0.0        -0.0000663   10
        0.0        0.05       0.0        1.0
            FINITE ELEMENT CHECK CASE


@ADD,P      J.RSAREP
```

```
@ASG,T    TP.,6C9,1962  .  TAPE WITH DATA & REDUCTION PROGRAM

@MOVE     TP,2
FURPUR 27R2  RL72R1 11/03/77 15:26:14

@COPIN    TP.,
 27 SYM  2 REL

@FREE     TP.

@ASG,T    15.,F14   .  SCRATCH FILE

@ASG,T    16.,F14   .  FILE WITH EXPERIMENTAL DATA FOR PLOTTING
```

@DATA,IL 15.
DATA T7 RL70-5 11/03-15:29:22

1.	4			
2.	38.0	2.0	18	
3.	5.0	0.0472	0.0676	14.0
4.	5.0	0.0476	0.068	14.0
5.	5.0	0.0459	0.0671	14.0
6.	10.0	0.0362	0.0402	26.0
7.	10.0	0.0352	0.0386	26.0
8.	10.0	0.036	0.0385	27.0
9.	15.0	0.0282	0.0317	39.0
10.	15.0	0.0279	0.0306	39.0
11.	15.0	0.0275	0.0303	39.0
12.	20.0	0.0241	0.026	49.0
13.	20.0	0.0238	0.0256	48.0
14.	20.0	0.0233	0.0253	48.0
15.	25.0	0.0213	0.0217	57.0
16.	25.0	0.0208	0.0214	58.0
17.	25.0	0.0206	0.0212	57.0
18.	30.0	0.0194	0.0182	73.0
19.	30.0	0.0189	0.0179	73.0
20.	30.0	0.0187	0.0175	73.0
21.	37.5	1.5	18	
22.	5.0	0.0714	0.143	
23.	5.0	0.0684	0.143	
24.	5.0	0.0684	0.143	
25.	10.0	0.0574	0.08	
26.	10.0	0.0538	0.0813	
27.	10.0	0.0515	0.0813	
28.	15.0	0.0424	0.0654	
29.	15.0	0.04	0.0621	
30.	15.0	0.0393	0.0629	
31.	20.0	0.0338	0.0508	
32.	20.0	0.0327	0.0488	
33.	20.0	0.0327	0.0485	
34.	25.0	0.0286	0.0435	
35.	25.0	0.0273	0.0413	
36.	25.0	0.0279	0.0417	
37.	30.0	0.0273	0.0368	
38.	30.0	0.0260	0.0357	
39.	30.0	0.0260	0.0355	
40.	39.0	3.0	18	
41.	5.0	0.0192	0.02	52.0
42.	5.0	0.0192	0.0192	52.0
43.	5.0	0.0192	0.0196	
44.	10.0	0.0151	0.0139	80.0
45.	10.0	0.0147	0.0137	80.0
46.	10.0	0.0142	0.0137	
47.	15.0	0.0116	0.0109	107.0
48.	15.0	0.0121	0.0106	105.0
49.	15.0	0.0121	0.0106	
50.	20.0	0.0104	0.0093	135.0
51.	20.0	0.0102	0.0089	135.0
52.	20.0	0.0100	0.0091	
53.	25.0	0.0092	0.0077	170.0
54.	25.0	0.0089	0.0075	170.0
55.	25.0	0.0086	0.0075	

56.	30.0	0.0080	0.0068	210.0
57.	30.0	0.0078	0.0067	205.0
58.	30.0	0.0076	0.0067	205.0
59.	39.5	3.5	18	
60.	5.0	0.0119	0.0168	82.0
61.	5.0	0.0119	0.0168	82.0
62.	5.0	0.0116	0.0168	82.0
63.	10.0	0.0093	0.0118	132.0
64.	10.0	0.0089	0.0119	132.0
65.	10.0	0.0086	0.0118	132.0
66.	15.0	0.0075	0.0093	170.0
67.	15.0	0.0074	0.0089	170.0
68.	15.0	0.0072	0.0088	170.0
69.	20.0	0.0065	0.0079	230.0
70.	20.0	0.0064	0.0076	230.0
71.	20.0	0.0063	0.0075	230.0
72.	25.0	0.0057	0.0068	265.0
73.	25.0	0.0056	0.0065	265.0
74.	25.0	0.0056	0.0065	265.0
75.	30.0	0.005	0.006	
76.	30.0	0.0050	0.0060	
77.	30.0	0.0050	0.0061	

END DATA.

```
@FOR,S   TPF$.NONDIM,N
FOR 00E3-11/03/77-15:31:29 (1,)

   MAIN PROGRAM

   STORAGE USED: CODE(1) 000156; DATA(0) 000135; BLANK COMMON(2) 000000

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   NINTR$
    0004   NRDU$
    0005   NIO2$
    0006   NWDU$
    0007   NWEF$
    0010   NSTOP$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0000   000023 1F       0000   000025 100F     0000   000102 101F     0000   000111 102F     0000   000100 103F
    0001   000025 113G     0001   000054 132G     0000   000076 2F       0001   000145 201L     0000   000021 3F
    0000   000022 4F       0001   000140 999L     0000 R 000000 C22B     0000 R 000012 FLEXI    0000 R 000011 FLEXO
    0000 I 000007 ID       0000 I 000002 IS       0000 I 000005 JDATA    0000 I 000001 JSETS    0000 R 000010 P
    0000 R 000014 PI       0000 R 000016 QFLEXI   0000 R 000017 QFLEXO   0000 R 000015 QNX      0000 R 000020 QWLOAD
    0000 R 000003 R        0000 R 000006 RHO      0000 R 000004 SR       0000 R 000013 WLOAD


00101          READ(5,3)  C22B                                                       000000
00105        3 FORMAT(F8.2)                                                          000007
00106          READ(15,4) JSETS                                                      000007
00111        4 FORMAT(I4)                                                            000020
00112          DO 201 IS=1,JSETS                                                     000020
00115          READ(15,1) R,SR,JDATA                                                 000025
00122        1 FORMAT (2F8.2,I4)                                                     000037
00123          RHO=R/SR                                                              000037
00124          WRITE(6,100) RHO,C22B                                                 000042
00130      100 FORMAT(1H1,// 10X,'NONDIMENSIONAL DATA FOR PRESSURE STABLIZED ARCH    000054
00130         1'    ///,5X'RADIUS RATIO='F10.4,/2X,'REFEEENCE RATIO='F10.4,//        000054
00130         215X,'NONDIMENSIONAL'/2X,'PRESSURE',6X'FLEXIBILITY',6X,'FLEXIBILITY    000054
00130         3',4X,'WRINKLING LOAD'/21X,'OUT',18X,'IN'/)                            000054
00131          DO 200 ID=1,JDATA                                                     000054
00134          READ(15,2,END=999) P,FLEXO,FLEXI,WLOAD                                000054
00142          PI=3.1415                                                             000066
00143        2 FORMAT(4F8.2)                                                         000070
00144          QNX=P*SR/(2.0*C22B)                                                   000070
00145          QFLEXI= FLEXI*PI*C22B                                                 000074
00146          QFLEXO=FLEXO*PI*C22B                                                  000077
00147          QWLOAD=WLOAD/(PI*SR*C22B)                                             000103
00150          WRITE(6,101) QNX,QFLEXO,QFLEXI,QWLOAD                                 000111
00156          WRITE(16,103) RHO,QNX,QFLEXO,QFLEXI,QWLOAD                            000122
00165      103 FORMAT(5E15.6)                                                        000136
00166      101 FORMAT(1X,E13.6,3X,E13.6,3X,E13.6,3X,E13.6)                           000136
00167      200 CONTINUE                                                              000136
00171          GO TO 201                                                             000136
```

```
00172      28*        999 WRITE(6,102)                                                          000140
00174      29*        102 FORMAT(1X,'END OF FILE ENCOUNTERED BEFORE ALL EXPECTED DATA WAS RE    000146
00174      30*           1AD')                                                                  000146
00175      31*        201 CONTINUE                                                              000146
00177      32*            END FILE 16                                                           000146
00200      33*            STOP                                                                  000151
00201      34*            END                                                                   000155

      END OF COMPILATION:         NO  DIAGNOSTICS.

@XQT
MAP28R1 RL71-3 11/03/77 15:32:57

ADDRESS LIMITS     001000 013735      5598 IBANK WORDS DECIMAL
                   040000 044202      2179 DBANK WORDS DECIMAL
STARTING ADDRESS   012142


              SEGMENT $MAIN$          001000 013735      040000 044202

NRWND$/FOR-E3            $(1)     001000 001063     $(2)     040000 040011
NFTCH$/FOR-E2            $(1)     001064 001346     $(2)     040012 040025
NBF00$                                              $(2)     040026 042253
NCNVT$/FOR68             $(1)     001347 001570     $(2)     042254 042350
NFTV$/FOR-E2             $(1)     001571 001613
NBDCV$/FOR-E0            $(1)     001614 001744     $(2)     042351 042426
NCLOS$/FOR-E3            $(1)     001745 002202     $(2)     042427 042454
NSWTC$/FOR69             $(1)     002203 002227
NWBLK$/FOR68             $(1)     002230 002341
NBSBL$/FOR68             $(1)     002342 002402
NUPDA$/FOR68             $(1)     002403 002436
NRBLK$/FOR-E2            $(1)     002437 002461
NOTIN$/FOR-E3            $(1)     002462 002756     $(2)     042455 042460
NININ/FOR-E3             $(1)     002757 003205     $(2)     042461 042466
NINPT$/FOR-E3            $(1)     003206 004575     $(2)     042467 042522
NFCHK$/FOR-E3            $(1)     004576 005567     $(2)     042523 042673
                         $(3)     005570 005570     $(4)     042674 042745
FORCOM$/FORFTN                                      $(2)     042746 042753
FORVCOM$/FOR-TE3                                    $(2)     042754 042763
ERU$/SYS72-8
NERCOM$/FOR-TE3          $(1)     005571 005650     $(2)     042764 042777
NIOER$/FOR-E3            $(1)     005651 006070     $(2)     043000 043147
NFMT$/FOR-E3             $(1)     006071 006753     $(2)     043150 043224
NOUT$/FOR-E3             $(1)     006754 010470     $(2)     043225 043266
NTAB$/FOR-TE3                                       $(2)     043267 043325
NSTOP$/FOR-TE3           $(1)     010471 010533     $(2)     043326 043335
NWEF$/FOR-E2             $(1)     010534 010741     $(2)     043336 043355
NOBUF$/FOR68             $(1)     010742 011002
NIBUF$/FOR-E2            $(1)     011003 011042     $(2)     043356 043356
NINTR$/FOR-E3            $(1)     011043 011115     $(2)     043357 043374
SQRT$/FOR59              $(1)     011116 011156     $(2)     043375 043406
NERR$/FOR-E3             $(1)     011157 011520     $(2)     043407 043566
```

```
NIER$/FOR-E3            $(1)     011521 011676     $(2)     043567 043706
NOSYM$/FOR-E3           $(1)     011677 012141     $(2)     043707 043710
BLANK$COMMON(COMMONBLOCK)
N                       $(1)     012142 012317     $(0)     043711 044045
                                                   $(2)     BLANK$COMMON
FPMIN                   $(1)     012320 013545     $(0)     044046 044127
                                                   $(2)     BLANK$COMMON
WRITEMATRIX             $(1)     013546 013735     $(0)     044130 044202
                                                   $(2)     BLANK$COMMON

SYS$*RLIB$. LEVEL 72-8
END MAP
```

NONDIMENSIONAL DATA FOR PRESSURE STABLIZED ARCH

RADIUS RATIO= 19.0000
REFEEENCE RATIO= 313.0000

PRESSURE	NONDIMENSIONAL FLEXIBILITY OUT	FLEXIBILITY IN	WRINKLING LOAD
.159744-01	.464113+02	.664704+02	.711896-02
.159744-01	.468046+02	.668637+02	.711896-02
.159744-01	.451330+02	.659787+02	.711896-02
.319489-01	.355951+02	.395282+02	.132209-01
.319489-01	.346118+02	.379550+02	.132209-01
.319489-01	.353984+02	.378566+02	.137294-01
.479233-01	.277288+02	.311703+02	.198314-01
.479233-01	.274338+02	.300887+02	.198314-01
.479233-01	.270405+02	.297937+02	.198314-01
.638978-01	.236973+02	.255655+02	.249164-01
.638978-01	.234023+02	.251722+02	.244079-01
.638978-01	.229106+02	.248772+02	.244079-01
.798722-01	.209441+02	.213374+02	.289843-01
.798722-01	.204524+02	.210424+02	.294928-01
.798722-01	.202558+02	.208457+02	.289843-01
.958466-01	.190758+02	.178959+02	.371203-01
.958466-01	.185842+02	.176009+02	.371203-01
.958466-01	.183875+02	.172076+02	.371203-01

NONDIMENSIONAL DATA FOR PRESSURE STABLIZED ARCH

RADIUS RATIO= 25.0000
REFEEENCE RATIO= 313.0000

PRESSURE	NONDIMENSIONAL FLEXIBILITY OUT	FLEXIBILITY IN	WRINKLING LOAD
.119808-01	.702069+02	.140610+03	.000000
.119808-01	.672570+02	.140610+03	.000000
.119808-01	.672570+02	.140610+03	.000000
.239617-01	.564408+02	.786632+02	.000000
.239617-01	.529010+02	.799414+02	.000000
.239617-01	.506394+02	.799414+02	.000000
.359425-01	.416915+02	.643071+02	.000000
.359425-01	.393316+02	.610623+02	.000000
.359425-01	.386433+02	.618489+02	.000000
.479233-01	.332352+02	.499511+02	.000000
.479233-01	.321536+02	.479845+02	.000000
.479233-01	.321536+02	.476895+02	.000000
.599042-01	.281221+02	.427731+02	.000000
.599042-01	.268438+02	.406099+02	.000000
.599042-01	.274338+02	.410032+02	.000000
.718850-01	.268438+02	.361851+02	.000000
.718850-01	.255655+02	.351034+02	.000000
.718850-01	.255655+02	.349068+02	.000000

NONDIMENSIONAL DATA FOR PRESSURE STABLIZED ARCH

RADIUS RATIO= 13.0000
REFEEENCE RATIO= 313.0000

PRESSURE	NONDIMENSIONAL FLEXIBILITY OUT	FLEXIBILITY IN	WRINKLING LOAD
.239617-01	.188792+02	.196658+02	.176279-01
.239617-01	.188792+02	.188792+02	.176279-01
.239617-01	.188792+02	.192725+02	.000000
.479233-01	.148477+02	.136677+02	.271199-01
.479233-01	.144544+02	.134711+02	.271199-01
.479233-01	.139627+02	.134711+02	.000000
.718850-01	.114062+02	.107179+02	.362728-01
.718850-01	.118978+02	.104229+02	.355948-01
.718850-01	.118978+02	.104229+02	.000000
.958466-01	.102262+02	.914459+01	.457648-01
.958466-01	.100296+02	.875128+01	.457648-01
.958466-01	.983289+01	.894793+01	.000000
.119808+00	.904626+01	.757133+01	.576297-01
.119808+00	.875128+01	.737467+01	.576297-01
.119808+00	.845629+01	.737467+01	.000000
.143770+00	.786632+01	.668637+01	.711896-01
.143770+00	.766966+01	.658304+01	.694946-01
.143770+00	.747300+01	.658804+01	.694946-01

NONDIMENSIONAL DATA FOR PRESSURE STABLIZED ARCH

RADIUS RATIO= 11.2857
REFEEENCE RATIO= 313.0000

PRESSURE	NONDIMENSIONAL FLEXIBILITY OUT	FLEXIBILITY IN	WRINKLING LOAD
.279553-01	.117011+02	.165193+02	.238267-01
.279553-01	.117011+02	.165193+02	.238267-01
.279553-01	.114062+02	.165193+02	.238267-01
.559105-01	.914459+01	.116028+02	.383552-01
.559105-01	.875128+01	.117011+02	.383552-01
.559105-01	.845629+01	.116028+02	.383552-01
.838658-01	.737467+01	.914459+01	.493969-01
.838658-01	.727634+01	.875128+01	.493969-01
.838658-01	.707968+01	.865295+01	.493969-01
.111821+00	.639138+01	.776799+01	.668311-01
.111821+00	.629305+01	.747300+01	.668311-01
.111821+00	.619472+01	.737467+01	.668311-01
.139776+00	.560475+01	.668637+01	.770010-01
.139776+00	.550642+01	.639138+01	.770010-01
.139776+00	.550642+01	.639138+01	.770010-01
.167732+00	.491645+01	.589974+01	.000000
.167732+00	.491645+01	.589974+01	.000000
.167732+00	.491645+01	.599807+01	.000000

@DATA,L 16.
DATA T7 RL70-5 11/03-15:34:52

1.	.190000+02	.159744-01	.464113+02	.664704+02	.711896-02
2.	.190000+02	.159744-01	.468046+02	.668637+02	.711896-02
3.	.190000+02	.159744-01	.451330+02	.659787+02	.711896-02
4.	.190000+02	.319489-01	.355951+02	.395262+02	.132209-01
5.	.190000+02	.319489-01	.346118+02	.379550+02	.132209-01
6.	.190000+02	.319489-01	.353984+02	.378566+02	.137294-01
7.	.190000+02	.479233-01	.277288+02	.311703+02	.198314-01
8.	.190000+02	.479233-01	.274338+02	.300887+02	.198314-01
9.	.190000+02	.479233-01	.270405+02	.297937+02	.198314-01
10.	.190000+02	.638978-01	.236973+02	.255655+02	.249164-01
11.	.190000+02	.638978-01	.234023+02	.251722+02	.244079-01
12.	.190000+02	.638978-01	.229106+02	.248772+02	.244079-01
13.	.190000+02	.798722-01	.209441+02	.213374+02	.289843-01
14.	.190000+02	.798722-01	.204524+02	.210424+02	.294928-01
15.	.190000+02	.798722-01	.202558+02	.208457+02	.289843-01
16.	.190000+02	.958466-01	.190758+02	.178959+02	.371203-01
17.	.190000+02	.958466-01	.185842+02	.176009+02	.371203-01
18.	.190000+02	.958466-01	.183875+02	.172076+02	.371203-01
19.	.250000+02	.119808-01	.702069+02	.140610+03	.000000
20.	.250000+02	.119808-01	.672570+02	.140610+03	.000000
21.	.250000+02	.119808-01	.672570+02	.140610+03	.000000
22.	.250000+02	.239617-01	.564408+02	.786632+02	.000000
23.	.250000+02	.239617-01	.529010+02	.799414+02	.000000
24.	.250000+02	.239617-01	.506394+02	.799414+02	.000000
25.	.250000+02	.359425-01	.416915+02	.643071+02	.000000
26.	.250000+02	.359425-01	.393316+02	.610623+02	.000000
27.	.250000+02	.359425-01	.386433+02	.618489+02	.000000
28.	.250000+02	.479233-01	.332352+02	.499511+02	.000000
29.	.250000+02	.479233-01	.321536+02	.479845+02	.000000
30.	.250000+02	.479233-01	.321536+02	.476895+02	.000000
31.	.250000+02	.599042-01	.281221+02	.427731+02	.000000
32.	.250000+02	.599042-01	.268438+02	.406099+02	.000000
33.	.250000+02	.599042-01	.274338+02	.410032+02	.000000
34.	.250000+02	.718850-01	.268438+02	.361851+02	.000000
35.	.250000+02	.718850-01	.255655+02	.351034+02	.000000
36.	.250000+02	.718850-01	.255655+02	.349068+02	.000000
37.	.130000+02	.239617-01	.188792+02	.196658+02	.176279-01
38.	.130000+02	.239617-01	.188792+02	.188792+02	.176279-01
39.	.130000+02	.239617-01	.188792+02	.192725+02	.000000
40.	.130000+02	.479233-01	.148477+02	.136677+02	.271199-01
41.	.130000+02	.479233-01	.144544+02	.134711+02	.271199-01
42.	.130000+02	.479233-01	.139627+02	.134711+02	.000000
43.	.130000+02	.718850-01	.114062+02	.107179+02	.362728-01
44.	.130000+02	.718850-01	.118978+02	.104229+02	.355948-01
45.	.130000+02	.718850-01	.118978+02	.104229+02	.000000
46.	.130000+02	.958466-01	.102262+02	.914459+01	.457648-01
47.	.130000+02	.958466-01	.100296+02	.875128+01	.457648-01
48.	.130000+02	.958466-01	.983289+01	.894793+01	.000000
49.	.130000+02	.119808+00	.904626+01	.757133+01	.576297-01
50.	.130000+02	.119808+00	.875128+01	.737467+01	.576297-01
51.	.130000+02	.119808+00	.845629+01	.737467+01	.000000
52.	.130000+02	.143770+00	.786632+01	.668637+01	.711896-01
53.	.130000+02	.143770+00	.766966+01	.658804+01	.694946-01
54.	.130000+02	.143770+00	.747300+01	.658804+01	.694946-01
55.	.112857+02	.279553-01	.117011+02	.165193+02	.238267-01

```
   56.         .112857+02     .279553-01     .117011+02     .165193+02     .238267-01
   57.         .112857+02     .279553-01     .114062+02     .165193+02     .238267-01
   58.         .112857+02     .559105-01     .914459+01     .116028+02     .383552-01
   59.         .112857+02     .559105-01     .875128+01     .117011+02     .383552-01
   60.         .112857+02     .559105-01     .845629+01     .116028+02     .383552-01
   61.         .112857+02     .838658-01     .737467+01     .914459+01     .493969-01
   62.         .112857+02     .838658-01     .727634+01     .875128+01     .493969-01
   63.         .112857+02     .838658-01     .707968+01     .865295+01     .493969-01
   64.         .112857+02     .111821+00     .639138+01     .776799+01     .668311-01
   65.         .112857+02     .111821+00     .629305+01     .747300+01     .668311-01
   66.         .112857+02     .111821+00     .619472+01     .737467+01     .668311-01
   67.         .112857+02     .139776+00     .560475+01     .668637+01     .770010-01
   68.         .112857+02     .139776+00     .550642+01     .639138+01     .770010-01
   69.         .112857+02     .139776+00     .550642+01     .639138+01     .770010-01
   70.         .112857+02     .167732+00     .491645+01     .589974+01     .000000
   71.         .112857+02     .167732+00     .491645+01     .589974+01     .000000
   72.         .112857+02     .167732+00     .491645+01     .599807+01     .000000
END DATA.

@ERS
FURPUR 27R2  RL72R1 11/03/77 15:34:56
END ERS.

@ERS     15.
END ERS.

@ASG,T   10.,F14    .  FILE WITH GAUSSIAN INTEGRATION CONSTANTS
```

```
@FOR,IS
FOR 00E3-11/03/77-15:42:08

   MAIN PROGRAM

   STORAGE USED: CODE(1) 000041; DATA(0) 000321; BLANK COMMON(2) 000000


   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   LGAUSS
    0004   NINTR$
    0005   NWDU$
    0006   NIO2$
    0007   NIO1$
    0010   NSTOP$


   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000012 1L        0001   000030 114G      0000   000312 2F        0000   000313 3F        0000 I 000311 I
    0000 I 000310 N         0000 R 000144 W         0000 R 000000 X



00101      1*            DIMENSION X(100),W(100)                                                     000000
00103      2*            N=90                                                                        000001
00104      3*            CALL LGAUSS(X,W,N,$1)                                                       000003
00105      4*      1     CONTINUE                                                                    000012
00106      5*            WRITE(10,2) N                                                               000012
00111      6*      2     FORMAT(I4)                                                                  000021
00112      7*            WRITE(10,3) (X(I),W(I),I=1,N)                                               000021
00121      8*      3     FORMAT(2E18.8)                                                              000034
00122      9*            STOP                                                                        000034
00123     10*            END                                                                         000040

       END OF COMPILATION:        NO  DIAGNOSTICS.


@XQT
MAP28R1 RL71-3 11/03/77 15:43:13


ADDRESS LIMITS     001000 010071      3642 IBANK WORDS DECIMAL
                   040000 044220      2193 DBANK WORDS DECIMAL
STARTING ADDRESS   010031



             SEGMENT $MAIN$         001000 010071      040000 044220

NSWTC$/FOR69            $(1)     001000 001024
NRBLK$/FOR-E2           $(1)     001025 001047
NRWND$/FOR-E3           $(1)     001050 001133     $(2)     040000 040011
```

```
NWEF$/FOR-E2           $(1)     001134 001341     $(2)     040012 040031
NBDCV$/FOR-EO          $(1)     001342 001472     $(2)     040032 040107
NFTV$/FOR-E2           $(1)     001473 001515
NCNVT$/FOR68           $(1)     001516 001737     $(2)     040110 040204
NCLOS$/FOR-E3          $(1)     001740 002175     $(2)     040205 040232
NWBLK$/FOR68           $(1)     002176 002307
NBSBL$/FOR68           $(1)     002310 002350
NUPDA$/FOR68           $(1)     002351 002404
NBF00$                                            $(2)     040233 042460
NOTIN$/FOR-E3          $(1)     002405 002701     $(2)     042461 042464
NOUT$/FOR-E3           $(1)     002702 004416     $(2)     042465 042526
NFMT$/FOR-E3           $(1)     004417 005301     $(2)     042527 042603
NIOER$/FOR-E3          $(1)     005302 005521     $(2)     042604 042753
NFCHK$/FOR-E3          $(1)     005522 006513     $(2)     042754 043124
                       $(3)     006514 006514     $(4)     043125 043176
NTAB$/FOR-TE3                                     $(2)     043177 043235
FORCOM$/FORFTN                                    $(2)     043236 043243
ERU$/SYS72-8
NERCOM$/FOR-TE3        $(1)     006515 006574     $(2)     043244 043257
FORVCOM$/FOR-TE3                                  $(2)     043260 043267
NERR$/FOR-E3           $(1)     006575 007136     $(2)     043270 043447
NSTOP$/FOR-TE3         $(1)     007137 007201     $(2)     043450 043457
NIER$/FOR-E3           $(1)     007202 007357     $(2)     043460 043577
NDBUF$/FOR68           $(1)     007360 007420
NINTR$/FOR-E3          $(1)     007421 007473     $(2)     043600 043615
LGAUSS                 $(1)     007474 010030     $(0)     043616 043677
                                                  $(2)     BLANK$COMMON
BLANK$COMMON(COMMONBLOCK)
NAMES                  $(1)     010031 010071     $(0)     043700 044220
                                                  $(2)     BLANK$COMMON

SYS$*RLIB$. LEVEL 72-8
END MAP
```

```
@DATA,L  10.
DATA T7 RL70-5 11/03-15:46:06
    1.          90
    2.             -.99964698+00       .90525975-03
    3.             -.99814039+00       .21074554-02
    4.             -.99543183+00       .33086729-02
    5.             -.99152394+00       .45059690-02
    6.             -.98642138+00       .56979301-02
    7.             -.98013026+00       .68829324-02
    8.             -.97265817+00       .80596809-02
    9.             -.96401411+00       .92266372-02
   10.             -.95420849+00       .10382556-01
   11.             -.94325311+00       .11525941-01
   12.             -.93116120+00       .12655360-01
   13.             -.91794731+00       .13769669-01
   14.             -.90362736+00       .14867329-01
   15.             -.88821861+00       .15947057-01
   16.             -.87173963+00       .17007603-01
   17.             -.85421026+00       .18047633-01
   18.             -.83565165+00       .19065906-01
   19.             -.81608614+00       .20061191-01
   20.             -.79553731+00       .21032342-01
   21.             -.77402992+00       .21978135-01
   22.             -.75158988+00       .22897445-01
   23.             -.72824423+00       .23789182-01
   24.             -.70402111+00       .24652231-01
   25.             -.67894970+00       .25485585-01
   26.             -.65306021+00       .26288226-01
   27.             -.62638382+00       .27059213-01
   28.             -.59895270+00       .27797571-01
   29.             -.57079988+00       .28502445-01
   30.             -.54195929+00       .29172971-01
   31.             -.51246569+00       .29808359-01
   32.             -.48235460+00       .30407821-01
   33.             -.45166232+00       .30970640-01
   34.             -.42042581+00       .31496140-01
   35.             -.38868273+00       .31983702-01
   36.             -.35647131+00       .32432720-01
   37.             -.32383037+00       .32842656-01
   38.             -.29079925+00       .33213020-01
   39.             -.25741773+00       .33543365-01
   40.             -.22372605+00       .33833297-01
   41.             -.18976479+00       .34082457-01
   42.             -.15557488+00       .34290551-01
   43.             -.12119751+00       .34457333-01
   44.             -.86674112-01       .34582590-01
   45.             -.52046277-01       .34666181-01
   46.             -.17355731-01       .34707999-01
   47.               .17355731-01       .34707999-01
   48.               .52046277-01       .34666181-01
   49.               .86674112-01       .34582590-01
   50.               .12119751+00       .34457333-01
   51.               .15557488+00       .34290551-01
   52.               .18976479+00       .34082457-01
   53.               .22372605+00       .33833297-01
   54.               .25741773+00       .33543365-01
   55.               .29079925+00       .33213020-01
```

```
   56.           .32383037+00       .32842656-01
   57.           .35647131+00       .32432720-01
   58.           .38868273+00       .31983702-01
   59.           .42042581+00       .31496140-01
   60.           .45166232+00       .30970640-01
   61.           .48235460+00       .30407821-01
   62.           .51246569+00       .29808359-01
   63.           .54195929+00       .29172971-01
   64.           .57079988+00       .28502445-01
   65.           .59895270+00       .27797571-01
   6·.           .62638382+00       .27059213-01
   67.           .65306021+00       .26288226-01
   68.           .67894970+00       .25485585-01
   69.           .70402111+00       .24652231-01
   70.           .72824423+00       .23789182-01
   71.           .75158988+00       .22897445-01
   72.           .77402992+00       .21978135-01
   73.           .79553731+00       .21032342-01
   74.           .81608614+00       .20061191-01
   75.           .83565165+00       .19065906-01
   76.           .85421026+00       .18047633-01
   77.           .87173963+00       .17007603-01
   78.           .88821861+00       .15947057-01
   79.           .90362736+00       .14867329-01
   80.           .91794731+00       .13769669-01
   81.           .93116120+00       .12655380-01
   82.           .94325511+00       .11525941-01
   83.           .95420849+00       .10382556-01
   84.           .96401411+00       .92266372-02
   85.           .97265817+00       .80596809-02
   86.           .98013026+00       .68829324-02
   87.           .98642138+00       .56979301-02
   88.           .99152394+00       .45059690-02
   89.           .99543183+00       .33086729-02
   90.           .99814039+00       .21074554-02
   91.           .99964698+00       .90525975-03
END DATA.

@ERS
FURPUR 27R2  RL72R1 11/03/77 15:46:15
END ERS.

@ASG,T    TP.,6C9,1964  .  TAPE WITH ARCH ANALYSIS PROGRAMS

@MOVE     TP,7
FURPUR 27R2  RL72R1 11/03/77 15:46:37

@COPIN    TP.,
 3 SYM

@FREE     TP.
```

```
@FOR,S    TPF$.MAINARCH.M
FOR 00E3-11/03/77-15:57:21 (0,)

   MAIN PROGRAM

   STORAGE USED: CODE(1) 003227; DATA(0) 004616; BLANK COMMON(2) 001541


   EXTERNAL REFERENCES (BLOCK, NAME)

    0003     PLOTS
    0004     NEWPEN
    0005     DGJR
    0006     CALCA
    0007     CALPAR
    0010     SCALE
    0011     AXIS
    0012     SYMBOL
    0013     NUMBER
    0014     PLOT
    0015     FLINE
    0016     LINE
    0017     EXIT
    0020     NINTR$
    0021     NRDC$
    0022     NIO2$
    0023     NRDU$
    0024     NIO3$
    0025     DSQRT
    0026     NPRT$
    0027     DSIN
    0030     DCOS
    0031     NREW$
    0032     NIO1$
    0033     NWEF$
    0034     NSTOP$


   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001     000027 10L      0000     004274 1001F      0000     004300 1002F      0000     004316 1003F      0000     004324 1004F
    0000     004335 1005F    0000     004346 1006F      0000     004220 1009F      0000     004216 1010F      0000     004352 1011F
    0000     004204 1012F    0000     004373 1013F      0000     004430 1014F      0000     004450 1015F      0000     004457 1016F
    0000     004464 1017F    0000     004471 1020F      0000     004175 1021F      0000     004200 1022F      0000     004160 1040F
    0000     004153 1041F    0000     004147 1042F      0001     002062 1105G      0001     002104 1122G      0001     002112 1132G
    0001     000125 20L      0000     004473 2001F      0001     001076 250L       0001     000411 251G       0001     001200 251L
    0001     001054 252L     0001     000415 255G       0001     001036 300L       0000     004475 3001F      0000     004476 3002F
    0000     004500 3003F    0000     004502 3004F      0000     004172 3005F      0000     004173 3006F      0001     001364 306L
    0001     000535 322G     0001     000537 325G       0001     001266 328L       0001     001335 329L       0001     000542 331G
    0001     000576 346G     0001     000605 353G       0001     000625 365G       0001     000626 370G       0001     001653 376L
    0001     001676 377L     0001     001733 395L       0001     001734 396L       0001     000144 40L        0001     000650 401G
    0001     002066 401L     0001     000651 404G       0001     000165 45L        0001     000755 460G       0001     000177 50L
    0001     002067 500L     0001     001040 507G       0001     002200 510L       0001     002334 518L       0001     002342 520L
    0001     002365 525L     0001     002375 530L       0001     002415 540L       0001     002417 550L       0001     002464 570L
    0001     002660 580L     0001     002740 600L       0001     001204 605G       0001     001205 610G       0001     002743 610L
    0001     001223 621G     0001     001224 624G       0001     001244 635G       0001     001245 640G       0001     003150 650L
    0001     001273 656G     0001     001313 670G       0001     001324 674G       0001     001353 710G       0001     001576 763G
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0001   001577 766G     0001   001610 776G     0001   003166 950L     0001   003161 960L     0001   003205 990L
0000 D 000032 A        0000 D 004050 AC       0000 D 004054 AE       0000 D 004035 ALPHA    0000 R 003307 AN
0000 R 003306 AR       0002 D 000007 ARR      0000 R 003310 AWI      0000 R 003311 AWO      0000 R 003312 AWRL
0000 D 000472 B        0000 D 004052 BC       0000 D 004056 BE       0002 D 000053 BETA     0002 D 000175 BLAM2
0002 D 000211 BLAM3    0000 D 004063 C        0000 R 001760 CN       0002 D 000111 CSA1     0002 D 000115 CSA3
0000 R 001612 CW       0000 R 002126 CWL      0002 D 000161 D        0002 D 000225 DA       0002 D 000005 E
0000 R 003140 ELN      0000 R 002624 EN       0000 R 002310 EWI      0000 R 002772 EWL      0000 R 002456 EWO
0000 D 004043 F        0002 D 000075 G        0002 D 000125 H        0002 D 000121 HCA2     0002 D 000123 HSA2
0000 I 004075 I        0002 I 001537 IBOUN    0002 I 000004 IC       0000 I 004102 ICL      0000 I 004034 IDUM
0000 I 004146 IEOF     0002 I 001540 IJUNC    0000 I 004033 ILOAD    0002 I 001535 INTEG    0000 I 004144 IP
0000 I 004032 IPLOT    0000 I 004141 ISAV     0002 I 001536 ITYP     0000 I 004077 J        0000 I 001576 JC
0000 I 004143 JP       0000 I 004076 K        0000 I 004106 KSET     0000 I 004103 LIN      0000 I 004145 NEXP
0000 I 004060 NIS      0000 I 004047 NMAX     0000 I 004142 NVAL     0000 D 004107 PA       0000 D 004111 PB
0000 D 004123 QCB      0000 D 004127 QCH      0000 D 004131 QG1      0000 D 004133 QG2      0000 D 004135 QG3
0000 D 004137 QG4      0000 D 000000 QIDEN    0000 D 004121 QSB      0000 D 004125 QSH      0002 D 000000 RHO
0000 D 004067 RHOSQ    0000 D 004071 RHOSQI   0000 D 003314 RLIM     0000 R 003313 RRHO     0000 D 004045 SMG
0002 D 000113 SNA1     0002 D 000117 SNA3     0002 D 000715 SOL      0000 D 001132 T        0000 D 003342 TARR
0000 D 004104 TEMPA    0000 D 004115 THEHAR   0000 D 004041 THEHAT   0000 D 004037 THETA    0000 D 004117 THETAR
0000 D 004113 THETR    0000 D 001572 V        0000 D 004100 WFA      0000 R 002304 WLSCAL   0000 R 002274 WSCAL
0000 D 003330 X        0000 D 004061 XC       0000 D 004065 XE       0002 D 000145 XI       0002 D 000002 XN
0000 D 003322 XNLIM    0000 R 002300 XNSCAL   0002 D 000153 XR       0002 D 000067 YLAM     0000 D 004073 YLAMSQ
0000 D 003334 Z
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00100    C                                                                             000000
00101          IMPLICIT DOUBLE PRECISION(A-H,O-Z)                                      000000
00101    C                                                                             000000
00101    C     ELASTIC ANALYSIS OF PRESSURE STABILIZED ARCH                            000000
0010·    C                                                                             000000
00101    C     REQUESTED BY EARL C STEEVES, GEPL   X2406                               000000
00101    C                                                                             000000
00101    C     PURPOSE - SOLUTION OF SET OF SECOND ORDER DIFFERENTIAL EQUATIONS        000000
00101    C                       WITH CONSTANT COEFFICIENTS                            000000
00101    C                                                                             000000
00101    C     C CHRETIEN                                                              000000
00101    C     OCTOBER 1973                                                            000000
00101    C        CHANGE   FEB 1974                                                    000000
00101    C                                                                             000000
00101    C                                                                             000000
00101    C                                                                             000000
00101    C     H=ARRAY CONTAINING CONSTANT COEFFICIENTS IN EQUATIONS                   000000
00101    C     E,RHO,C,XN - INPUT POSSIBLY VARYING OVER AN INTERVAL                    000000
00101    C          ARRAYS ELIM,RLIM,CLIM,XNLIM,ARE USED TO READ INIT,FINAL,DELTA      000000
00101    C              OF EACH                                                        000000
00101    C     XINT=ARRAY OF 3 VARIABLES DEFINED IN TERMS OF INTEGRAL                  000000
00101    C                              NUMERICALLY EVALUATED                          000000
00103          COMMON RHO,XN,IC,E                                                      000001
00104          COMMON ARR(3,6),BETA(6),YLAM(3),G(6),CSA1,SNA1,CSA3,SNA3                000001
00105          COMMON HCA2,HSA2,H(8)                                                   000001
00106          COMMON XI(3),XR(3),D(6),BLAM2(6),BLAM3(6)                               000001
00107          COMMON DA(12,13),SOL(200),INTEG,ITYP,IBOUN,IJUNC                        000001
00110          DIMENSION  QIDEN(13)                                                    000001
00111          DIMENSION A(12,12),B(12,12),T(12,12),V(2),JC(12)                        000001
00112          REAL CW(102),CN(102),CWL(102)                                           000001
00113          REAL WSCAL(4),XNSCAL(4),WLSCAL(4)                                       000001
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00114          REAL EWI(102),EWO(102),EN(102),EWL(102),ELN(102)          000001
00115         REAL AR,AN,AWI,AWO,AWRL,RRHO                               000001
00116         DIMENSION RLIM(3),XNLIM(3),X(2),Z(3)                       000001
00117         DIMENSION TARR(12,13)                                      000001
00120         READ  3001 IPLOT,ILOAD,INTEG,IBOUN                         000001
00120   C   IBOUN=0      CLAMPED ENDS                                    000001
00120   C    IBOUN=1     SIMPLY SUPPORTED ENDS                           000001
00120   C   INTEG=0 IMPLIES NO INTEGRATION OF GREEN S FUNCTION           000001
00120   C   INTEG=1 INPLIES INTEGRATION OF GREEN S FUNCTION              000001
00120   C   ITYP=0   CONCENTRATED LOAD AND INTEGRATION OF GREEN FUNCTION 000001
00120   C   ITYP=1   UNIFORM LOAD                                        000001
00120   C   ILOAD=0  IMPLIES NORMAL LOAD                                 000001
00120   C   ILOAD=1  IMPLIES VERTICAL LOAD                               000001
00126         IF (IPLOT .EQ. 1)  GO TO 10                                000011
00130         IDUM=100                                                   000014
00131         CALL PLOTS (IDUM,IDUM,20)                                  000016
00132         CALL NEWPEN (1)                                            000023
00132   C      CALL PLOT (0.0,-36.0,-3)                                  000023
00132   C      CALL PLOT (0.0,2.0,-3)                                    000023
00133      10 READ( 5,3002,END=990) RLIM,XNLIM                           000027
00137         READ 1001,ALPHA,THETA, THEHAT                              000042
00144         READ 2001, F,SMG,NMAX                                      000051
00151         READ 3002 AC,BC,AE,BE                                      000060
00157         NIS=0                                                      000070
00160         READ 1010,QIDEN                                            000071
00163         X(1)=0.0                                                   000100
00164         X(2)=2.0*3.14159                                           000102
00165         RHO=RLIM(1)                                                000104
00166         XC=AC*XNLIM(1)+BC                                          000106
00167         C=XC                                                       000112
00170         XE=AE*XNLIM(1)+BE                                          000113
00171         E=XE                                                       000117
00172         XN=XNLIM(1)                                                000120
00173         IC=0                                                       000122
00174         GO TO 50                                                   000123
00175      20 IF (RHO .GE. RLIM(2)) GO TO 10                             000125
00177         RHO=RHO+RLIM(3)                                            000130
00200         C=XC                                                       000133
00201         E=XE                                                       000135
00202         XN=XNLIM(1)                                                000137
00203         IC=0                                                       000141
00204         GO TO 50                                                   000142
00205      40 IF (XN .LT. XNLIM(2))  GO TO 45                            000144
00207         IF (SMG .EQ. 0.0 .OR. IPLOT .EQ. 1)  GO TO 20              000147
00211         GO TO 500                                                  000163
00212      45 XN=XN+XNLIM(3)                                             000165
00213         C=AC*XN+BC                                                 000167
00214         E=AE*XN+BE                                                 000172
00215      50 RHOSQ=RHO*RHO                                              000177
00216         RHOSQI=1.0/RHOSQ                                           000201
00217         Z(1)=RHOSQ*RHOSQ*(2.0/(SQRT(1.0-RHOSQI))-2.0-RHOSQI)       000204
00220         Z(2)=1.0+RHOSQI*Z(1)                                       000222
00221         Z(3)=(1.0+RHOSQI*(1.0-Z(1))+RHOSQI*RHOSQI*Z(1))            000225
00222         H(1)=2.0*E/RHO+E*Z(2)/(RHOSQ*RHO)                          000236
00223         H(2)=C*Z(3)/RHO+XN*Z(2)/RHO                                000251
00224         H(3)=E*Z(2)/RHOSQ                                          000261
00225         H(4)=C*Z(3)+XN*Z(1)/RHOSQ                                  000264
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00226          H(5)=H(1)+H(2)                                              000271
00227          H(6)=RHO*H(3)                                               000274
00230          H(7)=RHO*H(4)                                               000277
00231          H(8)=H(3)+H(4)                                              000302
00232          YLAMSQ=-(H(1)*H(4)*(H(4)-RHO*H(2)))/(H(2)*H(3)*(RHO*H(1)-H(3)))   000305
00233          YLAM(1)=SQRT(YLAMSQ)                                        000323
00234          YLAM(2)=0.0                                                 000327
00235          YLAM(3)=1.0                                                 000331
00235    C                                                                 000331
00235    C     CALCULATION OF BETA                                         000331
00235    C                                                                 000331
00235    C                                                                 000331
00235    C                                                                 000331
00236          A(1,1)=YLAMSQ*H(1)-H(2)                                     000333
00237          A(2,1)=YLAMSQ*H(3)-H(4)                                     000337
00240          A(1,2)=A(2,1)                                               000343
00241          A(2,2)=YLAMSQ*H(6)-H(7)                                     000344
00242      183 B(1,1)=-YLAM(1)*H(5)                                        000350
00243          B(2,1)=-YLAM(1)*H(8)                                        000354
00244          V(1)=1.0                                                    000360
00245          CALL DGJR(A,12,12,2,2,$960,JC,V)                            000361
00246          IF (JC(1) .NE. 2)  GO TO 950                                000373
00250          DO 190 I=1,2                                                000411
00253             BETA(I)=0.0                                              000411
00254             DO 185 K=1,2                                             000415
00257                BETA(I)=BETA(I)+A(I,K)*B(K,1)                         000415
00260      185    CONTINUE                                                 000421
00262             BETA(I)=-BETA(I)                                         000421
00263      190 CONTINUE                                                    000430
00265          A(1,1)=-H(1)-H(2)                                           000430
00266          A(2,1)=-H(3)-H(4)                                           000432
00267          A(3,1)=0.0                                                  000434
00270          A(4,1)=0.0                                                  000436
00271          A(1,2)=A(2,1)                                               000437
00272          A(2,2)=-H(6)-H(7)                                           000441
00273          A(3,2)=0.0                                                  000445
00274          A(4,2)=0.0                                                  000446
00275          A(1,3)=-2.0*H(1)                                            000447
00276          A(2,3)=-2.0*H(3)                                            000453
00277          A(3,3)=-H(1)-H(2)                                           000457
00300          A(4,3)=-H(3)-H(4)                                           000460
00301          A(1,4)=A(2,3)                                               000461
00302          A(2,4)=-2.0*H(6)                                            000463
00303          A(3,4)=A(2,1)                                               000467
00304          A(4,4)=A(2,2)                                               000471
00305          B(1,1)=H(5)                                                 000473
00306          B(2,1)=H(8)                                                 000475
00307          B(3,1)=0.0                                                  000477
00310          B(4,1)=0.0                                                  000500
00311          B(1,2)=-H(5)                                                000501
00312          B(2,2)=-H(8)                                                000503
00313          B(3,2)=-H(5)                                                000505
00314          B(4,2)=-H(8)                                                000507
00315          V(1)=1.0                                                    000511
00316          CALL DGJR(A,12,12,4,4,$960,JC,V)                            000513
00317          IF (JC(1) .NE. 4)  GO TO 950                                000525
00321          DO 200 I=1,4                                                000537
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00324          DO 200 J=1,2                                                   000537
00327             T(I,J)=0.0                                                  000537
00330          DO 200 K=1,4                                                   000542
00333             T(I,J)=T(I,J)+A(I,K)*B(K,J)                                 000542
00334      200 CONTINUE                                                       000561
00340          BETA(3)=-T(1,1)                                                000561
00341          BETA(4)=-T(2,1)                                                000563
00342          BETA(5)=-T(1,2)                                                000565
00343          BETA(6)=-T(2,2)                                                000567
00343    C                                                                    000567
00343    C     UNIFORM LOAD                                                   000567
00343    C                                                                    000567
00344          IJUNC=0                                                        000571
00345          DO 210 I=1,6                                                   000576
00350             BLAM2(I)=BETA(I)*YLAM(2)                                    000576
00351             BLAM3(I)=BETA(I)*YLAM(3)                                    000600
00352          DO 210 J=1,6                                                   000605
00355             A(I,J)=0.0                                                  000605
00356      210 CONTINUE                                                       000614
00361          IF (F .EQ. 0.0)  GO TO 300                                     000614
00363          CALL CALCA (ALPHA)                                             000616
00364          DO 212 I=1,3                                                   000626
00367             DO 211 J=1,6                                                000626
00372                TARR(I+3,J)=ARR(I,J)                                     000626
00373      211       CONTINUE                                                 000637
00375      212    CONTINUE                                                    000637
00377          CALL CALCA (-ALPHA)                                            000637
00400          DO 217 I=1,3                                                   000651
00403             DO 216 J=1,6                                                000651
00406                TARR(I,J)=ARR(I,J)                                       000651
0040       216       CONTINUE                                                 000662
00411      217    CONTINUE                                                    000662
00413          PRINT 1011                                                     000662
00415          IF(IBOUN.EQ.1)  PRINT 1041                                     000666
00420          IF(IBOUN.EQ.0)  PRINT 1042                                     000675
00423     1042 FORMAT(59X'CLAMPED ENDS')                                      000703
00424     1041 FORMAT(56X'SIMPLY SUPPORTED ENDS')                             000703
00425          PRINT 1010 QIDEN                                               000703
00430          PRINT 1040 AC,BC,AE,BE                                         000712
00436     1040 FORMAT(/5X,'C=',F6.4,'*N+',F6.4/5X,'E=',F6.3,'*N+',F6.4)       000722
00437          PRINT 1013, ALPHA,RHO,E,C,XN                                   000722
00446          PRINT 1014                                                     000733
00446    C                                                                    000733
00446    C     SOLVE FOR G                                                    000733
00446    C                                                                    000733
00450          G(1)=-F/H(1)                                                   000737
00451          TARR(1,7)=0.0                                                  000743
00452          TARR(2,7)=0.0                                                  000745
00453          TARR(3,7)=-G(1)                                                000746
00454          TARR(4,7)=0.0                                                  000747
00455          TARR(5,7)=0.0                                                  000750
00456          TARR(6,7)=-G(1)                                                000751
00456    C  UNIFORM LOAD IS REDUCED TO A PROBLEM OF ORDER 3 BY STMMETRY       000751
00457          DO 221 I=1,3                                                   000755
00462          TARR(I,2)=TARR(I,4)                                            000755
00463          TARR(I,3)=TARR(I,5)                                            000756
00464      221 TARR(I,4)=TARR(I,7)                                            000760
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00466          IF(IBOUN.EQ.1) TARR(2,4)= G(1)                 000763
00470          V(1)=4.0                                       000770
00471          CALL DGJR(TARR,13,12,3,4,$960,JC,V)            000772
00472          D(1)=TARR(1,4)                                 001004
00473          D(2)=0.0                                       001006
00474          D(3)=0.0                                       001010
00475          D(4)=TARR(2,4)                                 001011
00476          D(5)=TARR(3,4)                                 001013
00477          D(6)=0.0                                       001015
00500          ITYP=1                                         001016
0050           CALL CALPAR(-ALPHA,NMAX,THETA,WFA,0,0)         001020
00502          IF (SMG .EQ. 0.0)  GO TO 40                    001032
00502    C                                                    001032
00502    C     GREEN'S FUNCTION                               001032
00502    C     CONCENTRATED LOAD                              001032
00502    C                                                    001032
00504      300 CONTINUE                                       001040
00505     3005 FORMAT(I4)                                     001040
00506          DO  303  ICL=1,200                             001040
00511      303 SOL(ICL)=0.0                                   001040
00513          LIN=0                                          001042
00514          IF(INTEG.EQ.0) GO TO 250                       001043
00516          READ(10,3005) NIS                              001045
00521      252 LIN=LIN+1                                      001054
00522           IJUNC=0                                       001056
00523          READ(10,3006)  THETA,WFA                       001057
00527          THETA=THETA*ALPHA                              001066
00530          WFA=0.01745*ALPHA*WFA                          001071
00531     3006 FORMAT (2E18.8)                                001076
00532      250 CONTINUE                                       001076
00533          IF(INTEG.EQ.1) GOTO 251                        001076
00535          PRINT 1002                                     001100
00537          IF(ILOAD.EQ.0)PRINT 1021                       001104
00542          IF(ILOAD.EQ.1)PRINT 1022                       001112
00545     1021 FORMAT(61X,8H(NORMAL))                         001121
00546     1022 FORMAT(60X,10H(VERTICAL))                      001121
00547          IF(IBOUN.EQ.1)  PRINT 1041                     001121
00552          IF(IBOUN.EQ.0)  PRINT 1042                     001130
00555          PRINT 1010 QIDEN                               001136
00560          PRINT 1040 AC,BC,AE,BE                         001145
00566          PRINT 1013, ALPHA,RHO,E,C,XN                   001155
00575          PRINT 1003, THETA                              001166
00600          PRINT 1014                                     001173
00602      251 CONTINUE                                       001200
00603          G(1)=0.0                                       001200
00604          DO 310 I=1,3                                   001205
00607          DO 310 J=7,12                                  001205
00612             A(I,J)=0.0                                  001205
00613             A(I+3,J-6)=0.0                              001206
00614      310 CONTINUE                                       001214
00617          CALL CALCA(0.0)                                001214
00620          DO 320 I=1,3                                   001224
00623             DO 315 J=1,6                                001224
00626                A(I,J)=ARR(I,J)                          001224
00627      315       CONTINUE                                 001235
00631      320    CONTINUE                                    001235
00633          CALL CALCA(0.0)                                001235
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00634          DO 324 I=4,6                                            001245
00637             DO 323 J=7,12                                        001245
00642               A(I,J)=ARR(I-3,J-6)                                001245
00643    323        CONTINUE                                           001256
00645    324      CONTINUE                                             001256
00647          IJUNC=1                                                 001256
00650          TEMPA=THETA                                             001260
00651          KSET=1                                                  001262
00652          IF(KSET.EQ.1) GO TO 329                                 001263
00654    328   KSET=KSET+1                                             001266
00655          DO 360 J=1,6                                            001273
00660          A(10,J+6)=-A(10,J)                                      001273
00661          A(11,J+6)=-A(11,J)                                      001274
00662          A(12,J+6)=A(12,J)                                       001276
00663    360   CONTINUE                                                001301
00665          THETA=ALPHA+TEMPA                                       001301
00666          CALL CALCA(THETA)                                       001304
00667          DO 330 I=1,3                                            001313
00672          K=I+6                                                   001313
00673          DO 325 J=1,6                                            001316
00676          A(K,J)=ARR(I,J)                                         001324
00677    325   CONTINUE                                                001333
00701    330   CONTINUE                                                001333
00703          GO TO 306                                               001333
00704    329   CONTINUE                                                001335
00705          THETA=ALPHA-THETA                                       001335
00706          CALL CALCA(THETA)                                       001337
00707          DO 305 J=1,6                                            001353
00712          A(I,J+6)=-A(I,J)                                        001353
00713          A(7,J+6)=ARR(1,J)                                       001354
00714          A(8,J+6)=ARR(2,J)                                       001356
00715          A(9,J+6)=-ARR(3,J)                                      001360
00716    305   CONTINUE                                                001364
00720    304   CONTINUE                                                001364
00721    306   CONTINUE                                                001364
00722          PA=H(1)*(BETA(1)*YLAM(1)-1.0)+H(3)*BETA(2)*YLAM(1)      001364
00723          A(10,1)=PA*CSA1                                         001375
00724          A(10,2)=PA*SNA1                                         001377
00725          PA=BETA(1)*YLAM(1)-1.0+RHO*BETA(2)*YLAM(1)              001402
00726          A(11,1)=PA*CSA1                                         001407
00727          A(11,2)=PA*SNA1                                         001411
00730          PA=H(2)*(YLAM(1)+BETA(1))+H(4)*BETA(2)                  001414
00731          A(12,1)=PA*SNA1                                         001423
00732          A(12,2)=PA*CSA1                                         001425
00733          PA=(BETA(3)-1.0)*H(1)+H(3)*BETA(4)                      001430
00734          PB=(BETA(5)-BETA(3))*H(1)+H(3)*(BETA(6)-BETA(4))        001440
00735          THETR=THETA                                             001452
00736          THETA=0.01745*THETA                                     001454
00737          A(10,3)=PA*SNA3                                         001456
00740          A(10,4)=PA*CSA3                                         001460
00741          A(10,5)=PB*CSA3+THETA*A(10,3)                           001463
00742          A(10,6)=-PB*SNA3+THETA*A(10,4)                          001467
00743          PA=BETA(3)-1.0+RHO*BETA(4)                              001475
00744          PB=(BETA(5)-BETA(3))+RHO*(BETA(6)-BETA(4))              001501
00745          A(11,3)=PA*SNA3                                         001505
00746          A(11,4)=PA*CSA3                                         001507
00747          A(11,5)=PB*CSA3+THETA*A(11,3)                           001512
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00750          A(11,6)=-PB*SNA3+THETA*A(11,4)                  001517
00751          PA=H(2)*(1.0-BETA(3))-H(4)*BETA(4)              001525
00752          PB=H(2)*(1.0+BETA(5))+H(4)*BETA(6)              001535
00753          A(12,3)=PA*CSA3                                 001544
00754          A(12,4)=-PA*SNA3                                001547
00755          A(12,5)=A(12,3)*THETA+PB*SNA3                   001553
00756          A(12,6)=A(12,4)*THETA+PB*CSA3                   001560
00757          IF(KSET.EQ.1)  GO TO 328                        001566
00761          THETA=TEMPA                                     001571
00762          DO 370 I=1,12                                   001577
00765             DO 365 J=1,12                                001577
00770                DA(I,J)=A(I,J)                            001577
00771     365     CONTINUE                                     001610
00773     370     CONTINUE                                     001610
00775          DO 375 I=1,11                                   001610
01000             DA(I,13)=0.0                                 001610
01001     375     CONTINUE                                     001612
01003          THEHAR=0.01745*THEHAT                           001612
01004          THETAR=0.01745*THETA                            001615
01005          QSB=SIN(THETAR)                                 001620
01006          QCB=COS(THETAR)                                 001624
01007          QSH=SIN(THEHAR)                                 001630
01010          QCH=COS(THEHAR)                                 001634
01011          IF(ILOAD.EQ.1) GO TO 376                        001640
01013          QG1=0.0                                         001643
01014          QG2=0.0                                         001645
01015          QG3=SMG                                         001646
01016          QG4=0.0                                         001650
01017          GO TO 377                                       001651
01020     376 QG1=-SMG*QSH*QSH*QCB                             001653
0102           QG2=-SMG*QSB                                    001660
01022          QG3=SMG*QCH*QCH*QCB                             001664
01023          QG4=SMG*QCH*QSB                                 001672
01024     377 DA(10,13)=QG2                                    001676
01025          DA(11,13)=-QG4/H(3)                             001677
01026          DA(12,13)=QG1-QG3                               001703
01027          V(1)=4.0                                        001706
01030          CALL DGJR(DA,13,12,12,13,$960,JC,V)             001710
01031          G(1)=0.0                                        001722
01032          IF (IPLOT -1) 394,395,394                       001724
01035     394 ISAV=1                                           001727
01036          GO TO 396                                       001731
01037     395 ISAV=0                                           001733
01040     396 NVAL=0                                           001734
01041          ITYP=0                                          001734
01042          CALL CALPAR(-ALPHA,NMAX,THETA,WFA,ISAV,NVAL)    001735
01043          IF((LIN.LT.NIS).AND.(INTEG.EQ.1))  GO TO 252    001747
01045          IF(INTEG.EQ.0) GO TO 401                        001764
01047          REWIND  10                                      001766
01050          PRINT 1012                                      001771
01052          IF(IBOUN.EQ.1)  PRINT 1041                      001775
01055          IF(IBOUN.EQ.0)  PRINT 1042                      002004
01060          PRINT 1010,QIDEN                                002012
01063          PRINT 1040 AC,BC,AE,BE                          002021
01071          PRINT  1013,ALPHA,RHO,E,C,XN                    002031
01100          PRINT 1014                                      002042
01102          JP=8*(NMAX+1)                                   002046
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01103           PRINT 1009,((SOL(IP)),IP=1,JP)                                   002052
01111      1012 FORMAT('1',45X,'DEFORMATION OFPRESSURE STABILIZED ARCHES')       002066
01112      1010 FORMAT(1X,13A6)                                                  002066
01113       401 CONTINUE                                                         002066
01114      1009 FORMAT(/0PF10.3,1P7E17.8)                                        002066
01115           GO TO 40                                                         002066
01116       500 END FILE 15                                                      002067
01117           REWIND 15                                                        002071
01120           RRHO=RHO                                                         002074
01121           DO 503 I=1,4                                                     002104
0112·              WSCAL(I)=0.0                                                  002104
01125              XNSCAL(I)=0.0                                                 002104
01126              WLSCAL(I)=0.0                                                 002105
01127       503    CONTINUE                                                      002112
01131           DO 508 J=1,IC                                                    002112
01134           READ (15,3003) CN(J),CW(J),CWL(J)                                002112
01141           IF (CN(J) .LT. XNSCAL(1))  XNSCAL(1)=CN(J)                       002121
01143           IF (CN(J) .GT. XNSCAL(2))  XNSCAL(2)=CN(J)                       002127
01145           IF (CW(J) .LT. WSCAL(1))  WSCAL(1)=CW(J)                         002135
01147           IF (CW(J) .GT. WSCAL(2))  WSCAL(2)=CW(J)                         002143
01151           IF (CWL(J) .LT. WLSCAL(1))  WLSCAL(1)=CWL(J)                     002151
01153           IF (CWL(J) .GT. WLSCAL(2))  WLSCAL(2)=CWL(J)                     002157
01155       508 CONTINUE                                                         002167
01157           REWIND 15                                                        002167
01160           NEXP=0                                                           002172
01161           IEOF=0                                                           002173
01162           REWIND 16                                                        002174
01163       510 READ (16,3004,END=530) AR,AN,AWI,AWO,AWRL                        002200
01172           IF (ABS(AR-RRHO) .GT. 0.005)  GO TO 540                          002212
01174           NEXP=NEXP+1                                                      002220
01175           IF (AN .LT. XNSCAL(1))  XNSCAL(1)=AN                             002223
01177           IF (AN .GT. XNSCAL(2))  XNSCAL(2)=AN                             002231
01201           IF (AWI .LT. WSCAL(1) .AND. AWI .NE. 0.0)  WSCAL(1)=AWI          002237
01203           IF (AWO .LT. WSCAL(1) .AND. AWO .NE. 0.0)  WSCAL(1)=AWO          002254
01205           IF (AWI .GT. WSCAL(2) .AND. AWI .NE. 0.0)  WSCAL(2)=AWI          002271
01207           IF (AWO .GT. WSCAL(2) .AND. AWO .NE. 0.0)  WSCAL(2)=AWO          002306
01211           IF (AWRL .NE. 0.0)  GO TO 520                                    002323
01213           IF (NEXP .NE. 1)  GO TO 518                                      002325
01215           ELN(1)=0.0                                                       002330
01216           EWL(1)=0.0                                                       002331
01217           GO TO 525                                                        002332
01220       518 EWL(NEXP)=EWL(NEXP-1)                                            002334
01221           ELN(NEXP)=ELN(NEXP-1)                                            002336
01222           GO TO 525                                                        002340
01223       520 IF (AWRL .LT. WLSCAL(1))  WLSCAL(1)=AWRL                         002342
01225           IF (AWRL .GT. WLSCAL(2))  WLSCAL(2)=AWRL                         002351
01227           EWL(NEXP)=AWRL                                                   002357
01230           ELN(NEXP)=AN                                                     002362
01231       525 EN(NEXP)=AN                                                      002365
01232           EWO(NEXP)=AWO                                                    002367
01233           EWI(NEXP)=AWI                                                    002371
01234           GO TO 510                                                        002373
01235       530 IF (NEXP .NE. 0 .OR. IEOF .NE. 1)  GO TO 550                     002375
01237           IEOF=1                                                           002406
01240           REWIND 16                                                        002410
01241           GO TO 510                                                        002413
01242       540 IF (NEXP .EQ. 0)  GO TO 510                                      002415
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      550 CALL SCALE (XNSCAL,8.0,2,1)                                          002417
          CALL SCALE (WLSCAL,8.0,2,1)                                          002424
          CALL SCALE (WSCAL,8.0,2,1)                                           002432
          CN(IC+1)=XNSCAL(3)                                                   002440
          CN(IC+2)=XNSCAL(4)                                                   002443
          CW(IC+1)=WSCAL(3)                                                    002445
          CW(IC+2)=WSCAL(4)                                                    002447
          CWL(IC+1)=WLSCAL(3)                                                  002451
          CWL(IC+2)=WLSCAL(4)                                                  002453
          IF (NEXP .EQ. 0)  GO TO 570                                          002455
          EN(NEXP+1)=CN(IC+1)                                                  002457
          EN(NEXP+2)=CN(IC+2)                                                  002462
      570 CALL AXIS (0.0,0.0,'NONDIMENSIONAL PRESSURE',-23,5.0,0.0,CN(IC+1),   002464
         1           CN(IC+2))                                                 002464
          CALL AXIS (0.0,0.0,'NONDIMENSIONAL FLEXIBILITY',26,6.0,90.0,         002502
         1           CW(IC+1),CW(IC+2))                                        002502
          CALL SYMBOL (3.0,9.5,0.21,'RHO =',0.0,5)                             002521
          CALL NUMBER (4.5,9.5,0.21,RRHO,0.0,4)                                002531
          CALL NEWPEN (2)                                                      002541
          CALL PLOT (1.0,9.0,3)                                                002544
          CALL PLOT (1.5,9.0,2)                                                002551
          CALL SYMBOL (1.6,9.0,0.14,'CALC VAL',0.0,8)                          002556
          CALL FLINE (CN,CW,IC,1,0,0)                                          002566
          IF (NEXP .EQ. 0)  GO TO 600                                          002576
          CALL SCALE (EWI,8.0,NEXP,1)                                          002600
          IF (EWI(NEXP+1) .EQ. 0.0 .AND. EWI(NEXP+2) .EQ. 0.0)  GO TO 580      002606
          EWI(NEXP+1)=CW(IC+1)                                                 002620
          EWI(NEXP+2)=CW(IC+2)                                                 002622
          CALL NEWPEN (3)                                                      002624
          CALL LINE (EN,EWI,NEXP,1,-1,0)                                       002627
          CALL SYMBOL (3.605,9.07,0.14,0,0.0,-1)                               002637
          CALL SYMBOL (3.8,9.0,0.14,'EXP IN',0.0,6)                            002647
      580 CALL SCALE (EWO,8.0,NEXP,1)                                          002660
          IF (EWO(NEXP+1) .EQ. 0.0 .AND. EWO(NEXP+2) .EQ. 0.0)  GO TO 600      002665
          EWO(NEXP+1)=CW(IC+1)                                                 002677
          EWO(NEXP+2)=CW(IC+2)                                                 002701
          CALL NEWPEN (1)                                                      002703
          CALL SYMBOL (5.4,9.07,0.14,1,0.0,-1)                                 002706
          CALL SYMBOL (5.595,9.0,0.14,'EXP OUT',0.0,7)                         002716
          CALL LINE (EN,EWO,NEXP,1,-1,2)                                       002726
          GO TO 610                                                            002736
      600 CALL NEWPEN (1)                                                      002740
      610 CALL PLOT (10.0,0.0,-3)                                              002743
          CALL AXIS (0.0,0.0,'NONDIMENSIONAL PRESSURE',-23,5.0,0.0,CN(IC+1),   002747
         1           CN(IC+2))                                                 002747
          CALL AXIS (0.0,0.0,'NONDIMENSIONAL WRINKLING LOAD',29,6.0,90.0,      002766
         1           CWL(IC+1),CWL(IC+2))                                      002766
          CALL SYMBOL (3.0,9.5,0.21,'RHO =',0.0,5)                             003005
          CALL NUMBER (4.5,9.5,0.21,RRHO,0.0,4)                                003015
          CALL NEWPEN (2)                                                      003025
          CALL PLOT (1.0,9.0,3)                                                003030
          CALL PLOT (1.5,9.0,2)                                                003035
          CALL SYMBOL (1.6,9.0,0.14,'CALC VAL',0.0,8)                          003042
          CALL FLINE (CN,CWL,IC,1,0,0)                                         003052
          IF (NEXP .EQ. 0)  GO TO 650                                          003062
          CALL SCALE (EWL,8.0,NEXP,1)                                          003064
          IF (EWL(NEXP+1) .EQ. 0.0 .AND. EWL(NEXP+2) .EQ. 0.0)  GO TO 650      003072
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01337    488*          EWL(NEXP+1)=CWL(IC+1)                                          003104
01340    489*          EWL(NEXP+2)=CWL(IC+2)                                          003106
01341    490*          ELN(NEXP+1)=CN(IC+1)                                           003110
01342    491*          ELN(NEXP+2)=CN(IC+2)                                           003112
01343    492*          CALL NEWPEN (3)                                                003114
01344    493*          CALL LINE (ELN,EWL,NEXP,1,-1,5)                                003117
01345    494*          CALL SYMBOL (3.605,9.07,0.14,14,0.0,-1)                        003127
01346    495*          CALL SYMBOL (3.8,9.0,0.14,'EXP VAL',0.0,7)                     003137
01347    496*      650 CALL NEWPEN (1)                                                003150
01350    497*          CALL PLOT (10.0,0.0,-3)                                        003152
01351    498*          GO TO 20                                                       003157
01352    499*      960 PRINT 1016                                                     003161
01354    500*          GO TO 40                                                       003164
01355    501*      950 PRINT 1017                                                     003166
01357    502*          GO TO 40                                                       003171
01360   *DIAGNOSTIC*  CONTROL CAN NEVER REACH THE NEXT STATEMENT
01360    503*      970 PRINT 1004                                                     003172
01362    504*          GO TO 40                                                       003176
01363   *DIAGNOSTIC*  CONTROL CAN NEVER REACH THE NEXT STATEMENT
01363    505*      980 PRINT 1005                                                     003177
01365    506*          GO TO 40                                                       003203
01366    507*      990 PRINT 1006                                                     003205
01370    508*          IF (IPLOT .EQ. 1)  CALL EXIT                                   003210
01372    509*          CALL PLOT (4.0,0.0,999)                                        003215
01373    510*          CALL EXIT                                                      003222
01374    511*     1001 FORMAT (3F10.0,10X,3F10.0)                                     003226
01375    512*     1002 FORMAT ('1',45X,'DEFORMATION OF PRESSURE STABILIZED ARCHES'//  003226
01375    513*        1             57X,'CONCENTRATED LOAD')                           003226
01376    514*     1003 FORMAT (13X,'LOAD LOCATION =',F10.3)                           003226
01377    515*     1004 FORMAT ('1   ROOTS DO NOT CONVERGE IN 100 ITERATIONS')         003226
01400    516*     1005 FORMAT ('1   INTEGRAL DOES NOT CONVERGE IN 100 ITERATIONS')    003226
01401    517*     1006 FORMAT ('1   END OF JOB')                                      003226
01402    518*     1011 FORMAT ('1',45X,'DEFORMATION OF PRESSURE STABILIZED ARCHES'//  003226
01402    519*        1              49X,'NORMAL UNIFORM LOAD,DIRECT SOLUTION')        003226
01403    520*     1013 FORMAT (//11X,'HALF SPAN ANGLE =',F10.3/14X'RADIUS RATIO =',F10.3/  003226
01403    521*        1            5X,'ELASTIC MODULUS RATIO =',F10.3/                 003226
01403    522*        2            7X,'SHEAR MODULUS RATIO =',F10.3/                   003226
01403    523*        3            8X,'PRESSURE PARAMETER =',F10.3)                    003226
01404    524*     1014 FORMAT(///3X,'PSI',13X,'U',16X,'W',15X,'PHI',15X,'E(0)'        003226
01404    525*         112X,'E(PI)',11X,'N',17X,'M')                                   003226
01405    526*     1015 FORMAT ('   ERROR IN LAMBDA-SQ    ' ,6E18.8)                   003226
01406    527*     1016 FORMAT ('   OVERFLOW IN GJR  ')                                003226
01407    528*     1017 FORMAT ('   SINGULARITY IN GJR  ')                             003226
01410    529*     1020 FORMAT (7F10.3,I5)                                             003226
01411    530*     2001 FORMAT (2F10.0,I5)                                             003226
01412    531*     3001 FORMAT(4I1)                                                    003226
01413    532*     3002 FORMAT (7F10.0)                                                003226
01414    533*     3003 FORMAT (3E20.8)                                                003226
01415    534*     3004 FORMAT (5E15.6)                                                003226
01416    535*          END                                                            003226

        END OF COMPILATION:          2 DIAGNOSTICS.
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@FOR,S   TPF$.ARCHRESULTS,R
FOR 00E3-11/03/77-16:01:36 (0,)

   SUBROUTINE CALPAR     ENTRY POINT 001046

   STORAGE USED: CODE(1) 001075; DATA(0) 000222; BLANK COMMON(2) 001541

   E TERNAL REFERENCES (BLOCK, NAME)

    0003    DCOS
    0004    DSIN
    0005    DCOSH
    0006    DSINH
    0007    NPRT$
    0010    NIO2$
    0011    NWDU$
    0012    NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001     000750 10L        0001     001025 100L      0000     000101 1001F     0000     000104 1002F     0001     000764 11L
    0001     000150 116G       0001     000174 125G      0001     000201 3L        0001     000203 4L        0001     000222 5L
    0001     000702 6L         0002 D 000007 ARR         0002 D 000053 BETA        0002 D 000175 BLAM2       0002 D 000211 BLAM3
    0000 D 000075 CINT         0000 D 000024 CS          0002 D 000111 CSA1        0002 D 000115 CSA3        0000 D 000032 CSP
    0002 D 000161 D            0002 D 000225 DA          0000 D 000006 DELTH       0002 D 000005 E           0000 D 000052 EPI
    0000 D 000050 EZERO        0000 D 000002 FL          0002 D 000075 G           0002 D 000125 H           0002 D 000121 HCA2
    0002 D 000123 HSA2         0002     001537 IBOUN     0002 I 000004 IC          0000 I 000015 ID          0002     001540 IJUNC
    0000     000123 INJP$      0002 I 001535 INTEG       0002 I 001536 ITYP        0000 I 000010 N           0000 I 000074 NI
    0000 D 000044 PA           0000 D 000046 PB          0000 D 000060 PC          0000 D 000062 PD          0000 D 000064 PE
    0000 D 000066 PF           0000 D 000070 PG          0000 D 000072 PH          0000 D 000040 PHI         0000 D 000016 PSI
    0000 D 000013 PSIDEG       0000 D 000030 P1          0000 D 000022 P2          0000 D 000020 P3          0000 D 000054 QM
    0000 D 000056 QN           0000 D 000004 QNSTEP      0002 D 000000 RHO         0002 D 000002 SMN         0000 D 000026 SN
    0002 D 000113 SNA1         0002 D 000117 SNA3        0000 D 000034 SNP         0002 D 000715 SOL         0000 D 000036 U
    0000 D 000042 W            0000 D 000077 WRL         0002 D 000145 XI          0000 D 000011 XN          0002 D 000153 XR
    0000 D 000000 XXTH         0002 D 000067 YLAM


00101      1*          SUBROUTINE CALPAR (DEGREE,NSTEP,THETA,WFA,ISAV,IND)                      000000
00103      2*          IMPLICIT DOUBLE PRECISION(A-H,O-Z)                                       000000
00104      3*          COMMON RHO,SMN,IC,E                                                      000000
00105      4*          COMMON ARR(3,6),BETA(6),YLAM(3),G(6),CSA1,SNA1,CSA3,SNA3                 000000
00106      5*          COMMON HCA2,HSA2,H(8)                                                    000000
00107      6*          COMMON XI(3),XR(3),D(6),BLAM2(6),BLAM3(6)                                000000
00110      7*          COMMON DA(12,13),SOL(200),INTEG,ITYP,IBOUN,IJUNC                         000000
00111      8*           XXTH=.01745*THETA                                                       000000
00112      9*          FL=.0326*RHO/(RHO-1.)*COS(XXTH)                                          000002
00112     10*      C  FL CAN BE ENTERED AS A FUNCTION OF PSI                                    000002
00113     11*          QNSTEP=NSTEP                                                             000016
00114     12*          DELTH=-2.0*DEGREE/QNSTEP                                                 000024
00115     13*          DO 100 N=0,NSTEP                                                         000150
00120     14*          XN=N                                                                     000150
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00121        PSIDEG=    DEGREE+XN*DELTH                                    000156
00122        IF(ITYP.EQ.1)GO TO 5                                          000161
00124        DO  4   ID=1,6                                                000174
00127        IF(PSIDEG.GT.THETA)GO TO 3                                    000174
00131        D(ID)=DA(ID,13)                                               000175
00132        GO TO 4                                                       000177
00133      3 D(ID)=DA(ID+6,13)                                             000201
00134      4 CONTINUE                                                      000204
00136        PSI=(-DEGREE+PSIDEG)*0.01745                                  000204
00137        IF(PSIDEG.GT.THETA) PSI=(-DEGREE-PSIDEG)*0.01745              000210
00141      5 CONTINUE                                                      000222
00142        IF(ITYP .EQ.1) PSI=0.01745*PSIDEG                             000222
00144        P3=PSI*YLAM(3)                                                000227
00145        P2=PSI*YLAM(2)                                                000232
00146        CS=COS(P3)                                                    000235
00147        SN=SIN(P3)                                                    000241
00150        P1=PSI*YLAM(1)                                                000245
00151        CSP=COSH(P1)                                                  000250
00152        SNP=SINH(P1)                                                  000254
00153        U=D(1)*BETA(1)*SNP+D(2)*BETA(1)*CSP                           000260
00154        PHI=D(1)*BETA(2)*SNP+D(2)*BETA(2)*CSP                         000270
00155        W=D(1)*CSP+D(2)*SNP                                           000300
00156        PA=BETA(1)*YLAM(1)-1.0                                        000306
00157        PB=BETA(2)*YLAM(1)                                            000310
00160        EZERO=(PA-PB)*CSP*D(1)+(PA-PB)*SNP*D(2)                       000312
00161        EPI=(PA+PB)*CSP*D(1)+(PA+PB)*SNP*D(2)                         000325
00162        QM=-RHO*H(3)*(PA+RHO*PB)*(D(1)*CSP+D(2)*SNP)                  000336
00163        QN=(PA*RHO*H(1)+RHO*H(3)*PB)*(D(1)*CSP+D(2)*SNP)              000346
00164        U=U-D(3)*BETA(3)*CS+D(4)*BETA(3)*SN                           000356
00164       1     +D(5)*(BETA(5)*SN-BETA(3)*PSI*CS)                        000356
00164       2     +D(6)*(BETA(5)*CS+BETA(3)*PSI*SN)                        000356
00165        PHI=PHI-D(3)*BETA(4)*CS+D(4)*BETA(4)*SN                       000407
00165       1        +D(5)*(BETA(6)*SN-BETA(4)*PSI*CS)                     000407
00165       2        +D(6)*(BETA(6)*CS+BETA(4)*PSI*SN)                     000407
00166        W=W+D(3)*SN+D(4)*CS+D(5)*PSI*SN+D(6)*PSI*CS+G(1)              000440
00167        PA=BETA(3)-1.0-BETA(4)                                        000461
00170        PB=BETA(5)-BETA(3)-BETA(6)+BETA(4)                            000463
00171        PC=BETA(3)-1.0+BETA(4)                                        000465
00172        PD=BETA(5)-BETA(3)+BETA(6)-BETA(4)                            000467
00173        PE=(BETA(3)-1.0)*RHO*H(1)+RHO*H(3)*BETA(4)                    000471
00174        PF=(BETA(5)-BETA(3))*RHO*H(1)+RHO*H(3)*(BETA(6)-BETA(4))      000473
00175        PG=BETA(3)-1.0+RHO*BETA(4)                                    000475
00176        PH=BETA(5)-BETA(3)+RHO*(BETA(6)-BETA(4))                      000477
00177        EZERO=EZERO+D(3)*PA*SN+D(4)*PA*CS-G(1)                        000501
00177       1           +D(5)*(PB*CS+PA*PSI*SN)+D(6)*(-PB*SN+PA*PSI*CS)    000501
00200        EPI=EPI+D(3)*PC*SN+D(4)*PC*CS-G(1)                            000533
00200       1            +D(5)*(PD*CS+PC*PSI*SN)+D(6)*(-PD*SN+PC*PSI*CS)   000533
00201        EZERO=EZERO/(RHO+1.0)                                         000565
00202        EPI=EPI/(RHO-1.0)                                             000567
00203        QN=QN+D(3)*SN*PE+D(4)*PE*CS+D(5)*(PF*CS+PE*SN*PSI)+D(6)*(-PF  000571
00203       1*SN+PE*CS*PSI) -RHO*H(1)*G(1)                                 000571
00204        QM=QM-RHO*H(3)*(D(3)*PG*SN+D(4)*PG*CS+D(5)*(PH*CS+PG*SN*PSI)  000621
00204       1+D(6)*(-PH*SN+PG*CS*PSI)-G(1))                                000621
00205        QN=QN/RHO                                                     000653
00206        QM=QM/RHO                                                     000655
00207        IF((PSIDEG.LE.THETA).OR.(ITYP.EQ.1))  GO TO 6                 000660
00211        U=-U                                                          000675
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00212                PHI=-PHI                                                  000677
00213              6 CONTINUE                                                  000702
00214                IF(INTEG.EQ.0)GO TO 10                                    000702
00216                NI=8*N                                                    000703
00217                SOL(NI+1)=PSIDEG                                          000707
00220                CINT=WFA*FL                                               000711
00221                SOL(NI+2)=SOL(NI+2)+U*CINT                                000713
00222                SOL(NI+3)=SOL(NI+3)+W*CINT                                000716
00223                SOL(NI+4)=SOL(NI+4)+PHI*CINT                              000722
00224                SOL(NI+5)=SOL(NI+5)+EZERO*CINT                            000726
00225                SOL(NI+6)=SOL(NI+6)+EPI*CINT                              000732
00226                SOL(NI+7)=SOL(NI+7)+QN*CINT                               000736
00227                SOL(NI+8)=SOL(NI+8)+QM*CINT                               000742
00230                GO TO 11                                                  000746
00231             10 CONTINUE                                                  000750
00232                PRINT 1001, PSIDEG,U,W,PHI,EZERO,EPI,QN,QM                000750
00244             11 CONTINUE                                                  000764
00245                IF (ISAV .EQ. 0)  GO TO 100                               000764
00247                IF (ABS(PSIDEG) .GT. 0.5 .OR. IND .EQ. 1)  GO TO 100      000765
00251                WRL=ABS(SMN/EZERO)/E                                      001002
00252                WRITE (15,1002) SMN,W,WRL                                 001007
00257                IC=IC+1                                                   001017
00260                IND=1                                                     001022
00261            100 CONTINUE                                                  001026
00263                RETURN                                                    001026
00264           1001 FORMAT (/F10.3,1P7E17.8)                                  001074
00265           1002 FORMAT (3E20.8)                                           001074
00266                END                                                       001074

      END OF COMPILATION:        NO  DIAGNOSTICS.
```

```
@FOR,S   TPF$.BOUNDARYARCH,B
FOR 00E3-11/03/77-16:02:55 (0,)

   SUBROUTINE CALCA      ENTRY POINT 000250

   STORAGE USED: CODE(1) 000255; DATA(0) 000034; BLANK COMMON(2) 001541

   EXTERNAL REFERENCES (BLOCK, NAME)

    0003   DCOS
    0004   DSIN
    0005   DSINH
    0006   DCOSH
    0007   NERR3$

   STORAGE ASSIGNMENT  (BLOCK, TYPE, RELATIVE LOCATION, NAME)

    0001   000056 10L      0001   000073 11L      0001   000166 20L      0001   000226 21L      0000 D 000000 ANGLE
    0002 D 000007 ARR      0000 D 000004 A1       0000 D 000002 A3       0002 D 000053 BETA     0002 D 000175 BLAM2
    0002 D 000211 BLAM3    0000 D 000010 CON1     0000 D 000012 CON2     0000 D 000006 CO1      0002 D 000111 CSA1
    0002 D 000115 CSA3     0002 D 000161 D        0002 D 000225 DA       0002 D 000005 E        0002 D 000075 G
    0002 D 000125 H        0002 D 000121 HCA2     0002 D 000123 HSA2     0002 I 001537 IBOUN    0002   000004 IC
    0002 I 001540 IJUNC    0000   000020 INJP$    0002   001535 INTEG    0002   001536 ITYP     0002 D 000000 RHO
    0002 D 000113 SNA1     0002 D 000117 SNA3     0002 D 000715 SOL      0002 D 000145 XI       0002 D 000002 XN
    0002 D 000153 XR       0002 D 000067 YLAM

           SUBROUTINE CALCA (DEGREE)                                                000000
           IMPLICIT DOUBLE PRECISION(A-H,O-Z)                                       000000
           COMMON RHO,XN,IC,E                                                       000000
           COMMON ARR(3,6),BETA(6),YLAM(3),G(6),CSA1,SNA1,CSA3,SNA3                 000000
           COMMON HCA2,HSA2,H(8)                                                    000000
           COMMON XI(3),XR(3),D(6),BLAM2(6),BLAM3(6)                                000000
           COMMON DA(12,13),SOL(200),INTEG,ITYP,IBOUN,IJUNC                         000000
           ANGLE=0.01745*DEGREE                                                     000000
           A3=YLAM(3)*ANGLE                                                         000002
           CSA3=COS(A3)                                                             000004
           SNA3=SIN(A3)                                                             000010
           A1=YLAM(1)*ANGLE                                                         000014
           SNA1=SINH(A1)                                                            000017
           CSA1=COSH(A1)                                                            000023
           ARR(1,1)=BETA(1)*SNA1                                                    000027
           ARR(1,2)=BETA(1)*CSA1                                                    000032
           IF((IBOUN.EQ.1).AND.(IJUNC.EQ.0))  GO TO 10                              000034
           ARR(2,1)=BETA(2)*SNA1                                                    000046
           ARR(2,2)=BETA(2)*CSA1                                                    000051
           GO TO 11                                                                 000054
        10 CO1=BETA(1)*YLAM(1)-1.0+RHO*BETA(2)*YLAM(1)                              000056
           ARR(2,1)=CO1*CSA1                                                        000065
           ARR(2,2)=CO1*SNA1                                                        000067
        11 CONTINUE                                                                 000073
```

```
      ARR(3,1)=CSA1                                             000073
      ARR(3,2)=SNA1                                             000074
      ARR(1,3)=-BETA(3)*CSA3                                    000076
      ARR(1,4)=BETA(3)*SNA3                                     000102
      ARR(1,5)=BETA(5)*SNA3-BETA(3)*ANGLE*CSA3                  000105
      ARR(1,6)=BETA(5)*CSA3+BETA(3)*ANGLE*SNA3                  000116
      IF((IBOUN.EQ.1).AND.(IJUNC.EQ.0))  GO TO 20               000124
      ARR(2,3)=-BETA(4)*CSA3                                    000136
      ARR(2,4)=BETA(4)*SNA3                                     000142
      ARR(2,5)=BETA(6)*SNA3-BETA(4)*ANGLE*CSA3                  000145
      ARR(2,6)=BETA(6)*CSA3+BETA(4)*ANGLE*SNA3                  000156
      GO TO 21                                                  000164
   20 CON1=BETA(3)-1.0+RHO*BETA(4)                              000166
      CON2=BETA(5)-BETA(3)+RHO*(BETA(6)-BETA(4))                000173
      ARR(2,3)=CON1*SNA3                                        000202
      ARR(2,4)=CON1*CSA3                                        000204
      ARR(2,5)=CON2*CSA3+CON1*ANGLE*SNA3                        000207
      ARR(2,6)=-CON2*SNA3+CON1*ANGLE*CSA3                       000216
   21 CONTINUE                                                  000226
      ARR(3,3)=SNA3                                             000226
      ARR(3,4)=CSA3                                             000227
      ARR(3,5)=ANGLE*SNA3                                       000231
      ARR(3,6)=ANGLE*CSA3                                       000233
      RETURN                                                    000235
      END                                                       000254
```

END OF COMPILATION: NO DIAGNOSTICS.

```
@MAP,IN  A,B
MAP28R1 RL71-3 11/03/77 16:03:39 (,0)
END MAP

@ASG,T   20.,6C9,0928W  .  PLOT TAPE WITH APPROPRIATE NO.

@XQT   B
```

DEFORMATION OF PRESSURE STABILIZED ARCHES

CONCENTRATED LOAD
(NORMAL)
SIMPLY SUPPORTED ENDS

FINITE ELEMENT CHECK CASE

C= .0000*N+ .0500
E= .000*N+1.0000

HALF SPAN ANGLE =	90.000
RADIUS RATIO =	20.000
ELASTIC MODULUS RATIO =	1.000
SHEAR MODULUS RATIO =	.050
PRESSURE PARAMETER =	.057
LOAD LOCATION =	.000

PSI	U	W	PHI	E(0)	E(PI)	N	M
-90.000	-2.03287907-20	0.00000000	1 58475030-04	-1.65779550-05	-1.65779550-05	-3.31559099-05	4.23516474-23
-72.000	-3.25139188-04	-1.20542675-03	8.97332786-05	8.00368673-07	-3.95025582-05	-3.76939858-05	2.01388530-05
-54.000	-8.60732065-04	-1.24301006-03	-5.05428694-05	4.60105860-06	-4.43697882-05	-3.85436923-05	2.44701009-05
-36.000	-1.20240372-03	-7.94067007-06	-1.81650596-04	-1.15403683-06	-3.53225971-05	-3.56218854-05	1.70735891-05
-18.000	-9.79840769-04	2.18096021-03	-2.18952714-04	-2.19261753-05	-6.91271813-06	-2.92144648-05	-7.50203102-06
.000	4.74338450-20	4.50843540-03	-2.26089141-20	-7.90181502-05	6.26127394-05	-1.99483988-05	-7.07711297-05
18.000	9.79840769-04	2.18096021-03	2.18952714-04	-2.19261753-05	-6.91271813-06	-2.92144648-05	-7.50203102-06
36.000	1.20240372-03	-7.94067007-06	1.81650596-04	-1.15403683-06	-3.53225971-05	-3.56218854-05	1.70735891-05
54.000	8.60732065-04	-1.24301006-03	5.05428694-05	4.60105860-06	-4.43697882-05	-3.85436923-05	2.44701009-05
72.000	3.25139188-04	-1.20542675-03	-8.97332786-05	8.00368673-07	-3.95025582-05	-3.76939858-05	2.01388530-05
90.000	0.00000000	0.00000000	-1.58475030-04	-1.65779550-05	-1.65779550-05	-3.31559099-05	8.47032947-23

DEFORMATION OF PRESSURE STABILIZED ARCHES

CONCENTRATED LOAD
(NORMAL)
SIMPLY SUPPORTED ENDS

FINITE ELEMENT CHECK CASE

C= .0000*N+ .0500
E= .000*N+1.0000

HALF SPAN ANGLE = 90.000
RADIUS RATIO = 20.000
ELASTIC MODULUS RATIO = 1.000
SHEAR MODULUS RATIO = .050
PRESSURE PARAMETER = .067
LOAD LOCATION = .000

PSI	U	W	PHI	E(0)	E(PI)	N	M
-90.000	-1.01643954-20	0.00000000	1.36283977-04	-1.65779255-05	-1.65779255-05	-3.31558510-05	-1.48230766-22
-72.000	-2.96422217-04	-1.03989002-03	7.63284971-05	-1.55356452-06	-3.69647999-05	-3.76325295-05	1.76945378-05
-54.000	-7.72215360-04	-1.05442386-03	-4.59015665-05	1.69830150-06	-4.11982382-05	-3.84268521-05	2.14348479-05
-36.000	-1.07282052-03	3.92254658-05	-1.60391672-04	-3.00068467-06	-3.32162718-05	-3.54610941-05	1.50983394-05
-18.000	-8.73666662-04	1.97038106-03	-1.93859342-04	-2.06971344-05	-8.01096887-06	-2.90254559-05	-6.33911337-06
.000	-2.71050543-20	4.05357628-03	-2.41201731-20	-7.16352752-05	5.50548433-05	-1.97496669-05	-6.33054190-05
18.000	8.73666662-04	1.97038106-03	1.93859342-04	-2.06971344-05	-8.01096887-06	-2.90254559-05	-6.33911337-06
36.000	1.07282052-03	3.92254658-05	1.60391672-04	-3.00068467-06	-3.32162718-05	-3.54610941-05	1.50983394-05
54.000	7.72215360-04	-1.05442386-03	4.59015665-05	1.69830150-06	-4.11982382-05	-3.84268521-05	2.14348479-05
72.000	2.96422217-04	-1.03989002-03	-7.63284971-05	-1.55356452-06	-3.69647999-05	-3.76325295-05	1.76945378-05
90.000	0.00000000	0.00000000	-1.36283977-04	-1.65779255-05	-1.65779255-05	-3.31558510-05	1.05879118-22

DEFORMATION OF PRESSURE STABILIZED ARCHES

CONCENTRATED LOAD
(NORMAL)
SIMPLY SUPPORTED ENDS

FINITE ELEMENT CHECK CASE

C= .0000*N+ .0500
E= .000*N+1.0000

HALF SPAN ANGLE = 90.000
RADIUS RATIO = 30.000
ELASTIC MODULUS RATIO = 1.000
SHEAR MODULUS RATIO = .050
PRESSURE PARAMETER = .057
LOAD LOCATION = .000

PSI	U	W	PHI	E(0)	E(PI)	N	M
-90.000	1.62630326-19	4.65868121-21	2.19743349-04	-1.65779550-05	-1.65779550-05	-3.31559099-05	1.41172158-22
-72.000	-5.26901584-04	-2.02705377-03	1.23351702-04	-1.57546747-06	-3.67041428-05	-3.76939696-05	1.75594560-05
-54.000	-1.41243755-03	-2.11982231-03	-5.65979756-05	9.73197916-07	-4.02033255-05	-3.85436615-05	2.05825396-05
-36.000	-1.98626875-03	-9.85614664-05	-2.30614024-04	-1.81803947-06	-3.43460883-05	-3.56218429-05	1.62595041-05
-.3.000	-1.62621295-03	3.55609708-03	-3.08653672-04	-1.73204319-05	-1.18019832-05	-2.92144149-05	-2.75845749-06
.000	2.57498016-19	7.38875194-03	-8.99441353-19	-8.85435716-05	7.12593476-05	-1.99483463-05	-7.98792524-05
18.000	1.62621295-03	3.55609708-03	3.08653672-04	-1.73204319-05	-1.18019832-05	-2.92144149-05	-2.75845749-06
36.000	1.98626875-03	-9.85614664-05	2.30614024-04	-1.81803947-06	-3.43460883-05	-3.56218429-05	1.62595041-05
54.000	1.41243755-03	-2.11982231-03	5.65979756-05	9.73197916-07	-4.02033255-05	-3.85436615-05	2.05825396-05
72.000	5.26901584-04	-2.02705377-03	-1.23351702-04	-1.57546747-06	-3.67041428-05	-3.76939696-05	1.75594560-05
90.000	0.00000000	0.00000000	-2.19743349-04	-1.65779550-05	-1.65779550-05	-3.31559099-05	2.82344316-23

DEFORMATION OF PRESSURE STABILIZED ARCHES

CONCENTRATED LOAD
(NORMAL)
SIMPLY SUPPORTED ENDS

FINITE ELEMENT CHECK CASE

C= .0000*N+ .0500
E= .000*N+1.0000

HALF SPAN ANGLE = 90.000
RADIUS RATIO = 30.000
ELASTIC MODULUS RATIO = 1.000
SHEAR MODULUS RATIO = .050
PRESSURE PARAMETER = .067
LOAD LOCATION = .000

PSI	U	W	PHI	E(0)	E(PI)	N	M
-90.000	-2.43945489-19	2.54109884-21	1.87444223-04	-1.65779255-05	-1.65779255-05	-3.31558510-05	-3.10578747-22
-72.000	-4.74621036-04	-1.72613997-03	1.04025766-04	-3.83076676-06	-3.43098749-05	-3.76325154-05	1.52353185-05
-54.000	-1.25093116-03	-1.77526265-03	-5.06792375-05	-1.74853405-06	-3.72704883-05	-3.84268252-05	1.77560408-05
-36.000	-1.74888422-03	-8.60272085-06	-2.00449914-04	-3.80151175-06	-3.21318488-05	-3.54610570-05	1.41612316-05
-18.000	-1.43089193-03	3.17093732-03	-2.69625203-04	-1.65029362-05	-1.24549916-05	-2.90254123-05	-2.02340980-06
.000	-1.34847645-18	6.56372347-03	2.65411643-18	-7.97383034-05	6.23576056-05	-1.97496211-05	-7.10282080-05
18.000	1.43089193-03	3.17093732-03	2.69625203-04	-1.65029362-05	-1.24549916-05	-2.90254123-05	-2.02340980-06
36.000	1.74888422-03	-8.60272085-06	2.00449914-04	-3.80151175-06	-3.21318488-05	-3.54610570-05	1.41612316-05
54.000	1.25093116-03	-1.77526265-03	5.06792375-05	-1.74853405-06	-3.72704883-05	-3.84268252-05	1.77560408-05
72.000	4.74621036-04	-1.72613997-03	-1.04025766-04	-3.83076676-06	-3.43098749-05	-3.76325154-05	1.52353185-05
90.000	-2.03287907-20	0.00000000	-1.87444223-04	-1.65779255-05	-1.65779255-05	-3.31558510-05	2.82344316-23

SYMBOLS

A_i, B_i, C_i	constants of integration
b	force vector
a	cross-section radius
C_{ij}	material stiffness parameters
C	vector whose elements are the integration constants C_i
c,d	nondimensional material stiffnesses
E	nondimensional form of e and the energy density function
e	axial strain
F_i	applied surface loading
F_a	applied force
f_i	generalized forces associated with F_i
$\bar{f}_i$	nondimensional form of f_i
G_i	applied line load intensity
g_i	generalized forces associated with G_i
$\bar{g}_i$	nondimensional form of g_i
g	magnitude of concentrated load
H_i	coefficients of the governing differential equations
J_i	constants used in writing the boundary condition matrix
L_i	constants defined in Appendix E
l_g	gage length
M_x, M_y, M_z	nondimensional form of m_x, , m_y, m_z
m_x, m_y, m_z	internal moments

N_{ij}	stress resultants
N°_{ij}	stress resultants due to pressurization
N'_{ij}	first order stress resultants due to applied load
N''_{ij}	second order stress resultants due to applied load
n	nondimensional pressure parameter
P	pressure
Q_y, Q_z	nondimensional form of q_y, q_z
q_y, q_z	internal shear forces
R_1, R_2	principal radii of curvature
R	arch radius
S	coefficient matrix
s_{ij}	elements of matrix
s	arch length
T	nondimensional form of t
t	internal axial force
u_i	displacements due to applied load
U, V, W	cross-section displacements
$\mathcal{U}$, $\mathcal{V}$, $\mathcal{W}$	nondimensional form of U, V, W
v_i	general displacements
w_i	displacements due to pressurization
Z, Z_i	constants resulting from integration w.r.t. θ_1
α	half span angle of arch

α_2	parameter defining arch length
β_i	constants used in the relocations among A_i, B_i, and C_i
Γ_y, Γ_z	nondimensional form of γ_y, γ_z
γ_y, γ_z	transverse shear strains
γ	nondimensional flexibility
ϵ_{ij}	strain components
$\epsilon^{\circ}{}_{ij}$	strain components due to pressurization
$\epsilon'{}_{ij}$	first order strain components due to applied load
$\epsilon''{}_{ij}$	second order strain components due to applied load
θ_1, θ_2	coordinates locating position on arch
$\bar{\theta}_2$	location of applied line load
$\hat{\theta}_1$	location of concentrated load
K_x, K_y, K_z	nondimensional forms of κ_x, κ_y, κ_z
κ_x, κ_y, κ_z	curvatures
λ	parameter specifying the solution of the characteristic equation
ρ	radius ratio
ϕ_x, ϕ_y, ϕ_z	cross-section rotations
ω	angle defining region over which load acts and characteristic number

Subscript w denotes value parameter at wrinkling

$(\ \ )'$ denotes differentiation w.r.t. θ_2 except for ϵ_{ij} and N_{ij}

δ denotes variation